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In Optimal Control Problem in Economics with Four Linear Controls[1,2]

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Abstract. An optimal control problem with four linear controls describing a sophisticated concern model is investigated. The numerical solution of this problem by combination of a direct collocation and an indirect multiple shooting method is presented and discussed. The approximation provided by the direct method is used to estimate the switching structure caused by the four controls occurring linearly. The optimal controls have bang-bang subarcs as well as constrained and singular subarcs. The derivation of necessary conditions from optimal control theory is aimed at the subsequent application of an indirect multiple shooting method but is also interesting from a mathematical point of view. Due to the linear occurrence of the controls, the minimum principle leads to a linear programming problem. Therefore, the Karush-Kuhn-Tucker conditions can be used for an optimality check of the solution obtained by the indirect method.

Key Words. Microeconomic models, optimal control, linear controls, singular subarcs, necessary conditions, minimum principle as LP, direct collocation method, indirect multiple shooting method.

1. Introduction

Today, convenient and fast direct optimization methods enable the numerical solution of large and complicated optimal control problems. Moreover, a direct approach can be used to provide an initial trajectory for

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