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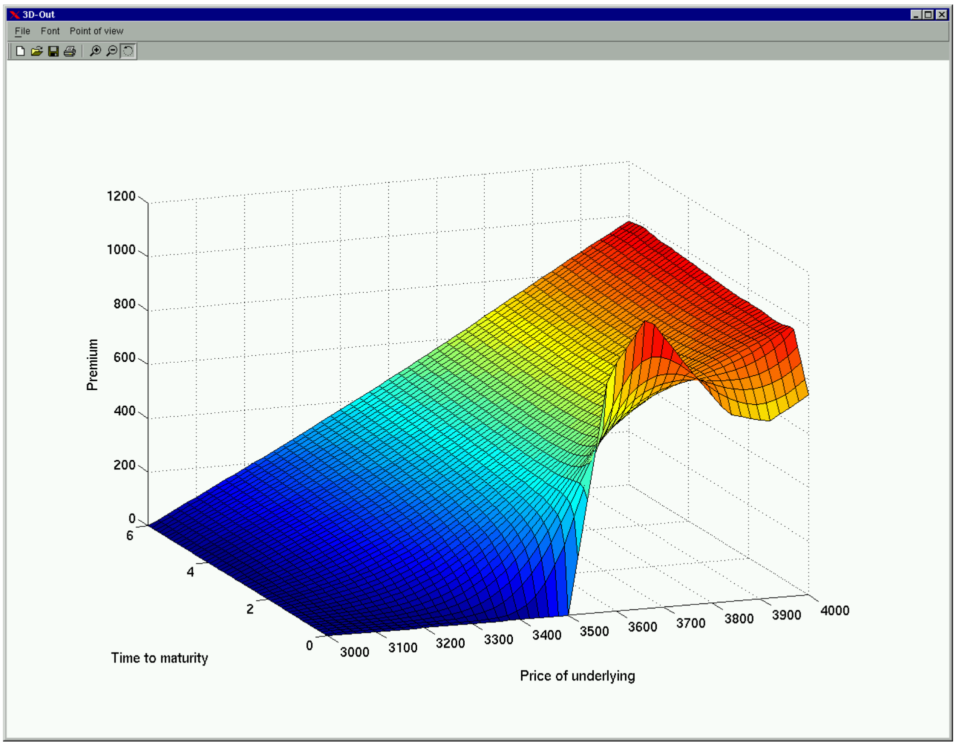
WARRANT-PRO-2: A GUI-Software for Easy Evaluation, Design and Visualization of European Double-Barrier Options [2]

Michael H. Breitner [3] and **Oliver Kubertin** [4]



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[1] Copies or a PDF-file are available upon request: Institut für Wirtschaftsinformatik, Universität Hannover, Königsworther Platz 1, D-30167 Hannover, Germany (www.iwi.uni-hannover.de).

[2] Paper submitted for the Proceedings of the OR 2003 – Annual International Conference of the German Operations Research Society (GOR), Universität Heidelberg, September 3 – 5, 2003. Attachment: PowerPoint slides of the talk presented at the OR 2003 and the first WARRANT-PRO-2 paper by M. H. Breitner and T. Burmester.

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Optimization of European Double-Barrier Options via Optimal Control of the Black-Scholes-Equation

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Abstract. In comparison to future contracts todays options have advantages and disadvantages. DAX-future contracts, e. g. the EUREX FDAX, have a constant profit/loss per DAX point, whereas the profit/loss per DAX point of common options varies undesirably with the DAX. An advantage of buying options results from the immediate payment of the option premium. There are no further payments during the life of the option. Similarly margin requirements force a FDAX buyer/seller to pay an initial margin plus a safety margin immediately. But in the event of adverse DAX movements the buyer/seller has to meet calls for substantial additional margin and may be forced to liquidate his position prematurely. Optimized European double-barrier options can combine the advantages of futures and options. The option expires if the DAX either hits the upper knock-out barrier $\mathrm{DAX}_{\max}$ or the lower knock-out barrier $\mathrm{DAX}_{\min}$ or if the option has matured. We present two approaches to optimize the cash settlements at expiration either analytically or numerically.

Keywords. Financial derivatives, options, futures, hedging tactics, Black-Scholes-model, optimal control, optimization, partial differential equation.

1 Real Life Case Study

A large German insurance company has a DAX-like stock portfolio worth 1 billion Euro (= 10^9 Euro). The DAX is 6000 points. "DAX" is used for the XETRA spot rate of the German stock index DAX 30. The company predicts decreasing stock values and wishes to immunize the portfolio, i. e. to generate a risk-free portfolio. Tax issues, market conditions or regulations inhibit the company from selling stocks.

2 Hedging with DAX Futures

The insurance company has the option to sell DAX futures at the EUREX, i. e. to open FDAX-short positions. One FDAX-short position has a profit/loss

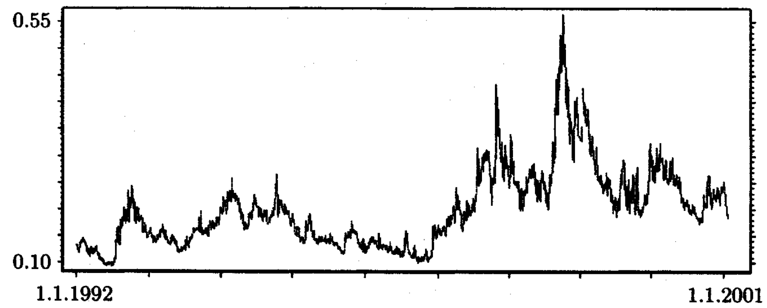

Fig. 1. Implied three-months at the money volatility (EUREX VDAX) of EUREX-options on the DAX. Source: Market-Maker (Release 1.40).

Δ_{FDAXs} per DAX point of 25 Euro. Thus the company has to sell $10^9/(25 * 6000) \approx 6667$ DAX futures to immunize the stock portfolio. The EUREX clearing agency demands an initial margin plus a safety margin immediately. The total margin per FDAX-short varies with the implied DAX volatility (EUREX VDAX) and is, e. g., about 5000 Euro for a low volatility of 0.17 and about 20000 Euro for a high volatility of 0.45. The total margin is between 3.33 % and 13.33 % of the stock portfolio value. In the event of an increasing DAX the company has to meet calls for additional margin, i. e. 166675 Euro per DAX point, and may be forced to liquidate its position prematurely. Moreover, the permanent margin adaption at the EUREX clearing agency is quite troublesome. Another disadvantage is the margin variation dependent on the VDAX, which oscillated between 0.10 and 0.55 during the last decade, see Fig. 1.

3 Hedging with DAX Calls with Constant Δ

The solution $C(t,p;r,\sigma)$ of the well known Black-Scholes-equation

$$\frac{\partial}{\partial t}C + r\,p\,\frac{\partial}{\partial p}C + \frac{1}{2}\sigma^2 p^2 \frac{\partial^2}{\partial p^2}C - r\,C = C_t + r\,p\,C_p + \frac{1}{2}\sigma^2 p^2 C_{pp} - r\,C \equiv 0, \quad (1)$$

which is a linear, homogeneous, parabolic partial differential equation of second order, yields a sufficiently accurate approximation of European DAX option values, see Hull (2000) and Redhead (1997). The time is denoted by $t \in [0,T]$, $p \in [\mathrm{DAX}_{\min}, \mathrm{DAX}_{\max}]$ denotes the DAX, $r > 0$ denotes the risk-free interest rate per year for the maturity period, $\sigma > 0$ denotes the implied volatility of the DAX and $T > 0$ denotes the initial maturity period at $t = 0$. Note that t and p are the independent variables in the linear, homogeneous, parabolic partial differential equation (1) of second order, whereas r and σ are constant parameters. For the profit/loss Δ per DAX point there holds $\Delta = C_p$. Assuming $\Delta \equiv \Delta_{\mathrm{opt}}$, $\Delta_{\mathrm{opt}} \geq 0$, for all

WARRANT-PRO-2: A GUI-Software for Easy Evaluation, Design and Visualization of European Double-Barrier Options

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Abstract. 2001 the first version WARRANT-PRO-2 (0.1) has been presented, see Breitner and Burmester (2002), which optimizes cash settlements for European double-barrier options and warrants. From the viewpoint of financial mathematics, some of the boundary conditions of the partial differential Black-Scholes equation are parameterized. The Black-Scholes equation is solved with a numerical Crank-Nicholson scheme and the parameters are optimized by nonlinear programming, i. e. an advanced SQP-method. In the upgraded version WARRANT-PRO-2 (0.2) an options deviation from a predefinable Delta (performance index) is minimized. The global error order of the Crank-Nicholson scheme is now quadratic in time (option's time to maturity) and space (market price of the option's underlying). The gradient of the performance index is computed highly accurate with automatic differentiation. Now a MATLAB-GUI (graphical user interface) allows easy evaluation, design and visualization of options and warrants. WARRANT-PRO-2 (0.2) and its GUI run stand-alone on LINUX PCs and laptops. Optimized options can combine the advantages of futures and options. Delta can be made almost constant for long periods and for a wide range of underlying market prices. Thus, no Delta-hedge adaptation is required. Moreover, tedious margining is not necessary. Optimized European double-barrier options are very interesting derivatives for both buyer and issuer and can revolutionize modern financial markets, see also www.iwi.uni-hannover.de/warrantpro2.html.

Keywords. Financial derivatives, options and futures, hedging tactics, Black-Scholes-model, optimal control and optimization, automatic differentiation, partial differential equations, software engineering and software quality.

1 Introduction

In comparison to future contracts todays options have advantages and disadvantages. *Exemplarily* the German stock index DAX 30 is taken as underlying. Further on "DAX" is used for the XETRA spot rate of the German stock index DAX 30. DAX-future contracts, e. g. the EUREX FDAX, have a constant profit/loss per DAX point, whereas the profit/loss per DAX point of common

WARRANT-PRO-2: A GUI-Software* for Easy Evaluation, Design and Visualization of European Double-Barrier Options

*** noncommercial**

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Topics

- **(Financial) Mathematics** and **Information Systems Research**;
- **European double-barrier options** (DAX calls);
- **Black-Scholes model** for options;
- A double-barrier DAX option **example** with a target;
- **Optimization** of European double-barrier options;
- **Software life cycle** and **software quality**;
- **WARRANT-PRO-2 Release 0.2** (2002 – 2003);
- **“Perfect” European double-barrier DAX call**;
- Other **examples**;
- **Software demonstration** (by Oliver Kubertin).