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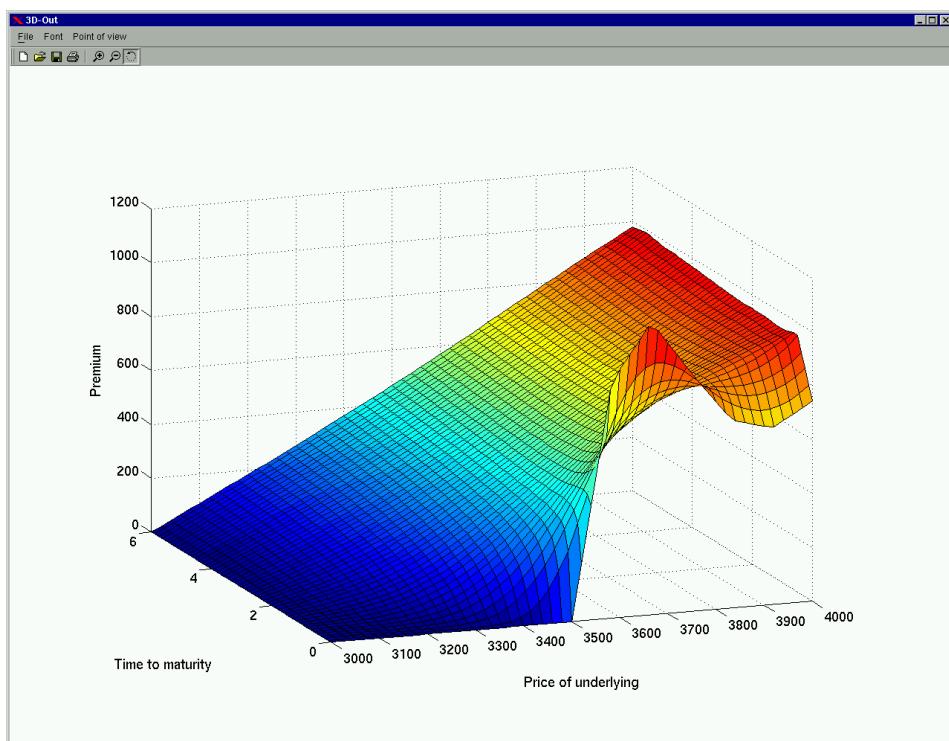
5 (September 12, 2003)¹



ISSN 1612-3646

WARRANT-PRO-2: A GUI-Software for Easy Evaluation, Design and Visualization of European Double-Barrier Options²

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² Paper submitted for the Proceedings of the OR 2003 – Annual International Conference of the German Operations Research Society (GOR), Universität Heidelberg, September 3 – 5, 2003. Attachment: PowerPoint slides of the talk presented at the OR 2003 and the first WARRANT-PRO-2 paper by M. H. Breitner and T. Burmester.

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WARRANT-PRO-2: A GUI-Software for Easy Evaluation, Design and Visualization of European Double-Barrier Options

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Abstract. 2001 the first version WARRANT-PRO-2 (0.1) has been presented, see Breitner and Burmester (2002), which optimizes cash settlements for European double-barrier options and warrants. From the viewpoint of financial mathematics, some of the boundary conditions of the partial differential Black-Scholes equation are parameterized. The Black-Scholes equation is solved with a numerical Crank-Nicholson scheme and the parameters are optimized by nonlinear programming, i. e. an advanced SQP-method. In the upgraded version WARRANT-PRO-2 (0.2) an options deviation from a predefinable Delta (performance index) is minimized. The global error order of the Crank-Nicholson scheme is now quadratic in time (option's time to maturity) and space (market price of the option's underlying). The gradient of the performance index is computed highly accurate with automatic differentiation. Now a MATLAB-GUI (graphical user interface) allows easy evaluation, design and visualization of options and warrants. WARRANT-PRO-2 (0.2) and its GUI run stand-alone on LINUX PCs and laptops. Optimized options can combine the advantages of futures and options. Delta can be made almost constant for long periods and for a wide range of underlying market prices. Thus, no Delta-hedge adaptation is required. Moreover, tedious margining is not necessary. Optimized European double-barrier options are very interesting derivatives for both buyer and issuer and can revolutionize modern financial markets, see also www.iwi.uni-hannover.de/warrantpro2.html.

Keywords. Financial derivatives, options and futures, hedging tactics, Black-Scholes-model, optimal control and optimization, automatic differentiation, partial differential equations, software engineering and software quality.

1 Introduction

In comparison to future contracts todays options have advantages and disadvantages. *Exemplarily* the German stock index DAX 30 is taken as underlying. Further on “DAX” is used for the XETRA spot rate of the German stock index DAX 30. DAX-future contracts, e. g. the EUREX FDAX, have a constant profit/loss per DAX point, whereas the profit/loss per DAX point of common

options varies undesirably with the DAX. An advantage of buying options results from the immediate payment of the option premium. There are no further payments during the life of the option. Similarly margin requirements force a FDAX buyer/seller to pay an initial margin plus a safety margin immediately. But in the event of adverse DAX movements the buyer/seller has to meet calls for substantial additional margin and may be forced to liquidate his position prematurely, see Breitner and Burmester (2002) for details.

Optimized European double-barrier options can combine the advantages of futures and options. An European double-barrier option expires if the DAX either hits the upper knock-out barrier DAX_{\max} or the lower knock-out barrier DAX_{\min} or if the option has matured.

2 Black-Scholes-Equation

The solution $C(t, p; r, \sigma)$ of the well known Black-Scholes-equation

$$\frac{\partial}{\partial t}C + rp\frac{\partial}{\partial p}C + \frac{1}{2}\sigma^2p^2\frac{\partial^2}{\partial p^2}C - rC = C_t + rpC_p + \frac{1}{2}\sigma^2p^2C_{pp} - rC \equiv 0, \quad (1)$$

which is a linear, homogeneous, parabolic partial differential equation of second order, yields a sufficiently accurate approximation of European DAX option values, see Hull (2003) and Redhead (1997). The time is denoted by $t \in [0, T]$, $p \in [DAX_{\min}, DAX_{\max}]$ denotes the DAX, $r > 0$ denotes the risk-free interest rate per year for the maturity period, $\sigma > 0$ denotes the implied volatility of the DAX and $T > 0$ denotes the initial maturity period at $t = 0$. Note that t and p are the independent variables in equation (1), whereas r and σ are constant parameters. The most important option greek Delta (symbol: Δ) is the first partial derivative of $C(t, p; r, \sigma)$ w. r. t. p . For an *European double-barrier* DAX call the boundary conditions at $p = DAX_{\min}$, $t = T$ and $p = DAX_{\max}$ correspond to the cash settlements at the option's expiration. The option expires if either one of the knock-out barriers is hit, i. e. $p = DAX_{\max}$ at the upper barrier or $p = DAX_{\min}$ at the lower barrier, or if the expiration date is reached, i. e. $t = T$.

3 WARRANT-PRO-2 (0.2)

The development of WARRANT-PRO-2 started in 1999. Version 0.1 was completed at the end of 2000, see Breitner and Burmester (2002). Primary goal is to optimize cash settlements for European double-barrier options and warrants. From the viewpoint of financial mathematics, the boundary conditions of the partial differential Black-Scholes equation are parameterized. Then the Black-Scholes equation is solved with a numerical Crank-Nicholson scheme and the parameters are optimized by nonlinear programming, i. e. by the advanced sequential quadratic programming (SQP) method NPSOL,

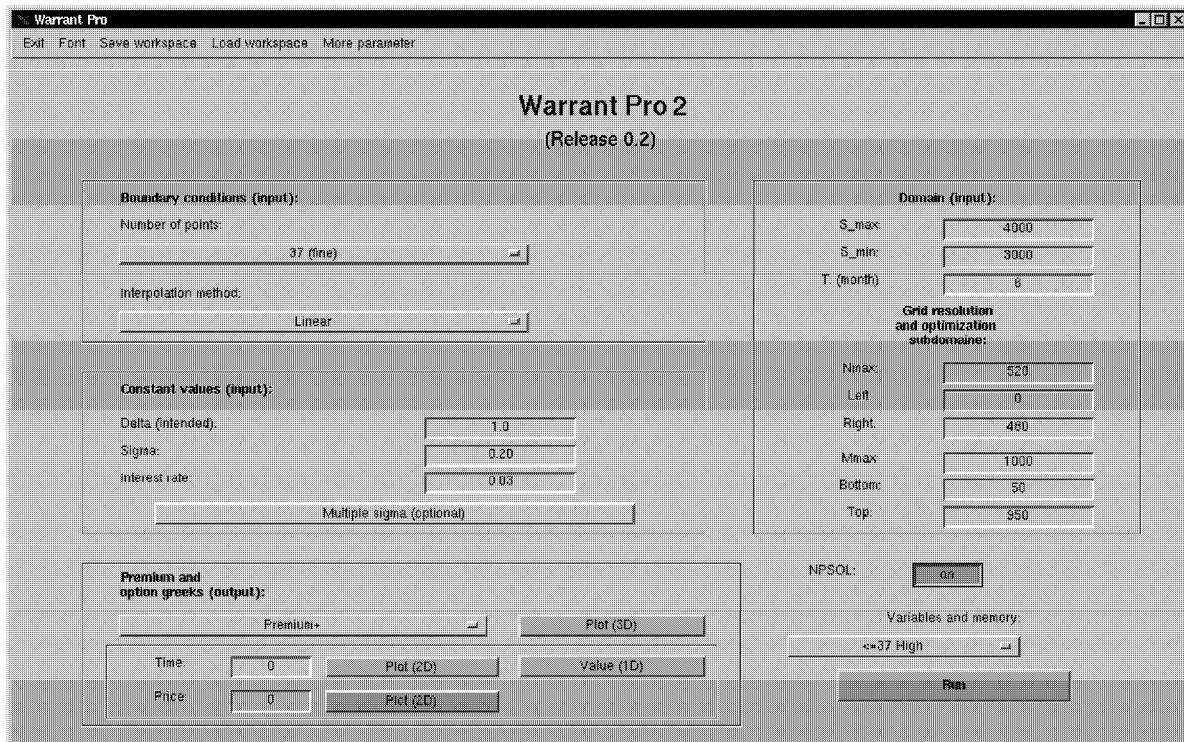


Fig. 1. GUI main window to control the inputs boundary conditions' parameterization and interpolation, Δ_{opt} , single or multiple σ , r , output 1-d., 2-d. and 3-d. graphics (left), and inputs evaluation domain, optimization domain, grid, optimization or simulation and RAM allocation (right).

see Gill et al. (2000). The performance index — to be minimized — is the options deviation from a predefinable Δ_{opt} for a set of up to 8 volatilities, see Breitner and Burmester (2002).

The development of version 0.2 started in spring 2002. In the upgraded version various improvements are made and new features are implemented. The kernel is still coded in ANSI FORTRAN 77 to assure portability, see www.fortran.com for ANSI FORTRAN standards. Public domain and free-ware ANSI FORTRAN 77 compilers are available for *all* computer platforms, see www.fortran.com, too.

3.1 ANSI FORTRAN 77 Kernel

An upgraded Crank-Nicholson scheme is used, which is a combination of an explicit and an implicit finite difference method, see Ames (1994), Hull (2003), Prisman (2000), Seydel (2000) and Thomas (1998). With a time discretization Δt and DAX discretization Δp the upgraded scheme is more accurate. The absolute, global error of the numerical solution decreases faster by $\mathcal{O}(\Delta t^2) + \mathcal{O}(\Delta p^2)$ for $\max(\Delta t, \Delta p) \rightarrow 0$. The convergence of the upgraded Crank-Nicholson scheme for all relevant t , p , r and σ has been proven by the second author in his diploma thesis.

The main memory (RAM) allocation is reduced by the choice of 9 different, precompiled modes. Dependent on the fineness of the Crank-Nicholson



Fig. 2. GUI window for a fine boundary conditions' parameterization with 37 equidistant points. Box constraints for all parameters can individually limit or freeze parameters during the optimization.

grid and the fineness of the boundary conditions' parameterization the user can limit the RAM allocation. For the parameterization 10 (rough), 19 (moderate) and 37 (fine) equidistant points are available, see Fig. 1 upper left. Different interpolation methods between the points are selectable: Linear, (cubic) spline and (cubic) Hermite-spline, see Fig. 1 upper left, too.

The gradient of the performance index is needed in order to minimize the performance index with the SQP method NPSOL. The gradient is now computed highly accurate, i. e. with the Crank-Nicholson scheme's accuracy. *Automatic* differentiation — also called algorithmic differentiation — instead of numerical differentiation is used, see Griewank (2000). Automatic differentiation is concerned with the accurate and efficient evaluation of derivatives for functions defined by computer programs. No truncation errors are incurred and the resulting numerical derivative values can be used for all scientific computations. Chainrule techniques for evaluating derivatives of composite functions are used. Automatic differentiation today is applied to larger and larger programs, e. g. for optimum shape design. In many such applications modeling was restricted to simulations at various parameters settings. Today, with the help of automatic differentiation techniques, this trial and error approach can be replaced by a more efficient optimization w. r. t. modes, design and control parameters, see also Hoffmann et al. (1999) and Lions (1971). Alternatively computer algebra systems, e. g. MAPLE, see www.maplesoft.com, can be used to calculate analytic derivatives for functions. Here, based on the ANSI FORTRAN 77 code for the computation

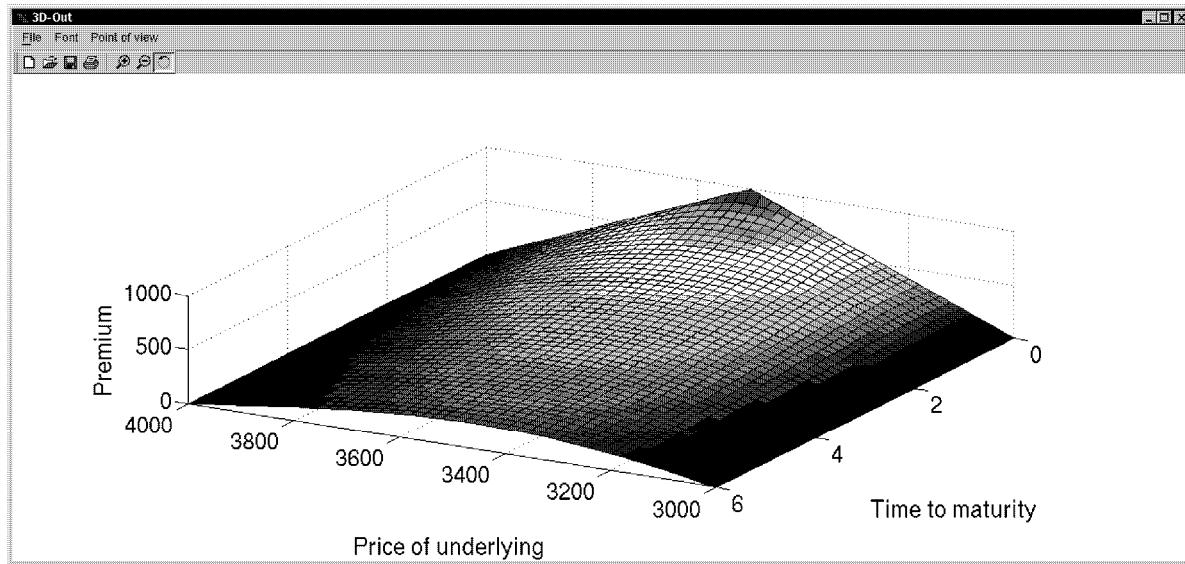


Fig. 3. Black-Scholes value of an European double-barrier option computed by WARRANT-PRO-2 (simulated). The option has a target of 3500 DAX points, i. e. the cash settlements are 0 Euro on the barriers, $(p - 3000)*2$ Euro and $(4000 - p)*2$ Euro, respectively.

of the performance index efficient ANSI FORTRAN 77 code for the gradient computation has been generated. For the code generation ADIFOR 2.0, see www.cs.rice.edu/~adifor, has been used. The gradient of the performance index can be computed relatively cheap in comparison to the numerical differentiation used in WARRANT-PRO-2 (0.1).

Summarized the new ANSI FORTRAN 77 kernel is about 10 to 20 times faster compared to the kernel of version 0.1. The robustness of the convergence of the SQP method NPSOL has improved significantly due to the significantly higher accuracy of the gradient. The reduced RAM allocation enables the usage of older computers with only 32 or 64 MB RAM. Moreover smaller problems can be stored completely in large level-2 or level-3 cache memories (≥ 4 MB). If so, an additional speed up of 2.5 to 3 is gained.

3.2 GUI — Graphical User Interface

Modern software engineering techniques focus on software quality. Todays software quality requirements primarily include efficiency, maintainability, portability, functionality, reliability and user friendliness. On the one hand, programming a user friendly GUI is very import for modern software, on the other hand the GUI programming usually is very time consuming and often a mandatory GUI reduces portability. A well-designed GUI should be intuitively obvious to the user. Providing an interface between user and application, GUIs enable users to operate the application without knowing commands and/or formats required by a command line and/or file interface. Thus applications providing GUIs are easier to learn and easier to use.

Tools for semi-automatic and fast setup of GUIs — also called rapid application development (RAD) tools — are a compromise, e. g. Borland

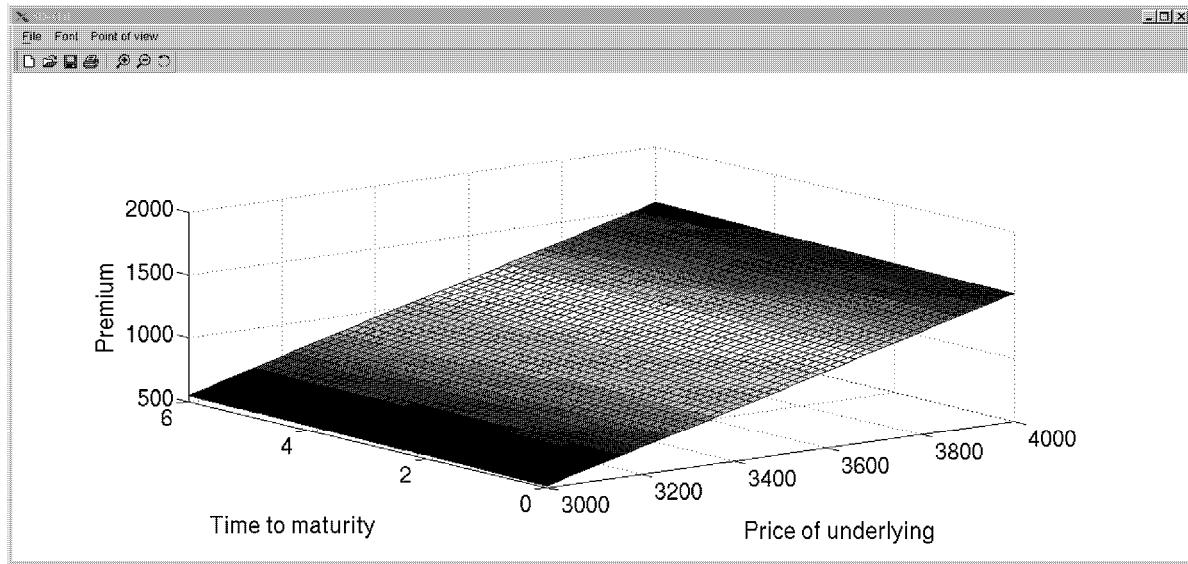


Fig. 4. Black-Scholes value of an European double-barrier option computed by WARRANT-PRO-2 (optimized). All parameters are unconstrained and WARRANT-PRO-2 accurately computes the solution, compare the analytic solution calculated in Breitner and Burmester (2002).

Delphi 7 Studio Enterprise, see www.borland.com/delphi, or MATLAB Release 13, see www.mathworks.com. For the WARRANT-PRO-2 (0.2) GUI the MATLAB GUIDE and and MATLAB Compiler have been used to generate C and C++ code. The C and C++ code has been compiled and linked with the MATLAB run-time libraries. A stand-alone executable for LINUX PCs and laptops was built which calls the compiled kernel program. The stand-alone application runs even if MATLAB is not installed on the end-user's system. Internal information flows via input and output files. The file concept has the important advantage that the compiled kernel program of the version 0.2 runs *without* GUI, too. Remote logins on LINUX compute servers are possible for time-consuming computations. A stand-alone application for WINDOWS computers can be compiled and linked analogously.

The GUI has several windows: The main window, see Fig. 1, the boundary conditions' window, see Fig. 2, the multiple σ window and the 1-dimensional, 2-dimensional and 3-dimensional graphics windows, see Figs. 3 – 6. High quality screenshots and details are available from the first authors WWW-page www.iwi.uni-hannover.de/warrantpro2.html.

4 Examples

Unless originally designed to optimize cash settlements, WARRANT-PRO-2 (0.2) proved to be very useful and comfortable also for the computation of Black-Scholes values *without* optimization, see Fig. 3 for an option with a target. The most important option greeks can be evaluated and plotted easily, too. Optimized options are depicted in Figs. 4–6 and are explained in

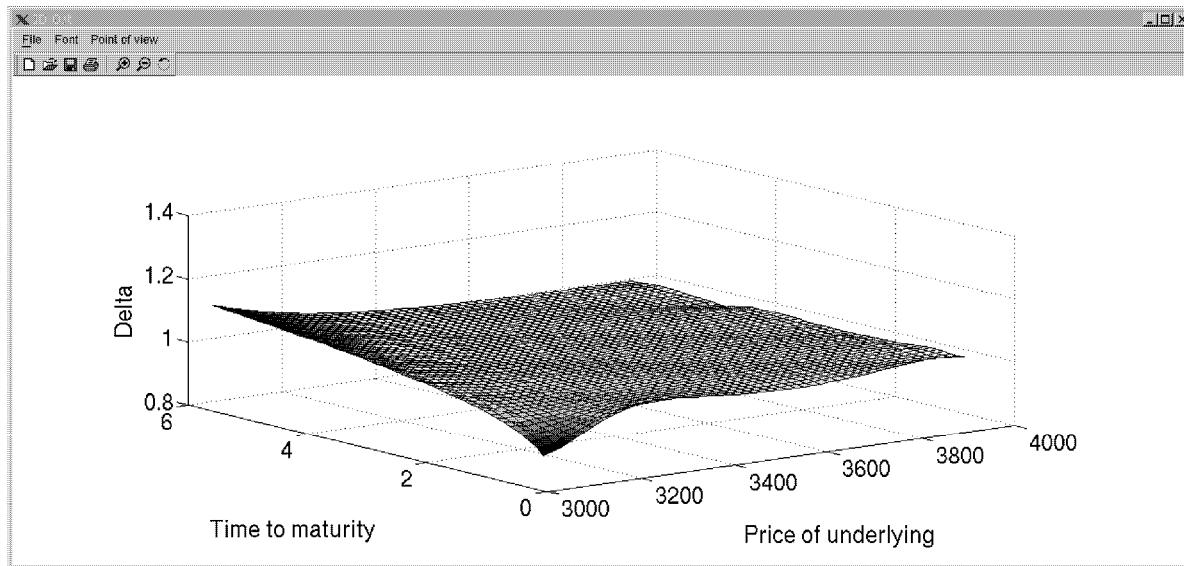


Fig. 5. Black-Scholes Δ of an European double-barrier option computed by WARRANT-PRO-2 (optimized). The optimization domain is $3050 \leq p \leq 3950$ and $6 \text{ months} \leq t \leq 0.5 \text{ months}$. The cash settlement is 0 Euro for $p = 3000$ DAX points and the other cash settlements are optimized.

the figure captions. In Fig. 4 a very interesting solution with $\Delta \equiv 1$ is shown which also can be calculated analytically, see Breitner and Burmester (2002).

5 Conclusions

European double-barrier options are very flexible and powerful options which can revolutionize modern financial markets. WARRANT-PRO-2, currently an *uncommercial* software, proved to be an ideal tool to optimize the cash settlements and to visualize an option's Black-Scholes value and option greeks. Moreover, for given cash settlements an option's value and the option greeks can be computed fast and comfortably. Interested readers can contact the authors to get a *freeware* trial version of WARRANT-PRO-2 (0.2).

Acknowledgment

The authors gratefully appreciate support by Prof. Dr. P. E. Gill, University of California San Diego, providing the excellent SQP optimization method NPSOL, and by Prof. Dr. Michael W. Fagan, Rice University, Houston, providing the excellent automatic differentiation package ADIFOR 2.0.

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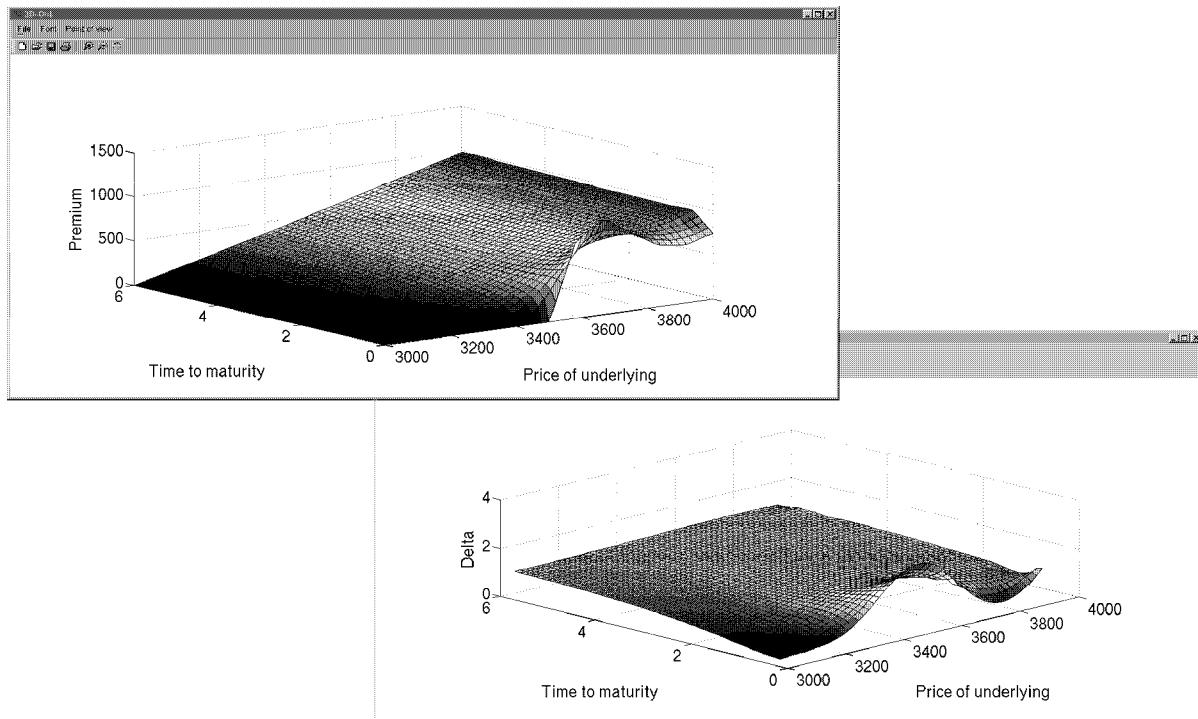


Fig. 6. Black-Scholes value and Δ of an European double-barrier option computed by WARRANT-PRO-2 (optimized). The optimization domain is $3050 \leq p \leq 3950$ and $6 \text{ months} \leq t \leq 0.5 \text{ months}$. The cash settlement is 0 Euro for $p \leq 3500$ DAX points and the other cash settlements are optimized, compare to Figs. 3 and 6.

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WARRANT-PRO-2: A GUI-Software* for Easy Evaluation, Design and Visualization of European Double-Barrier Options

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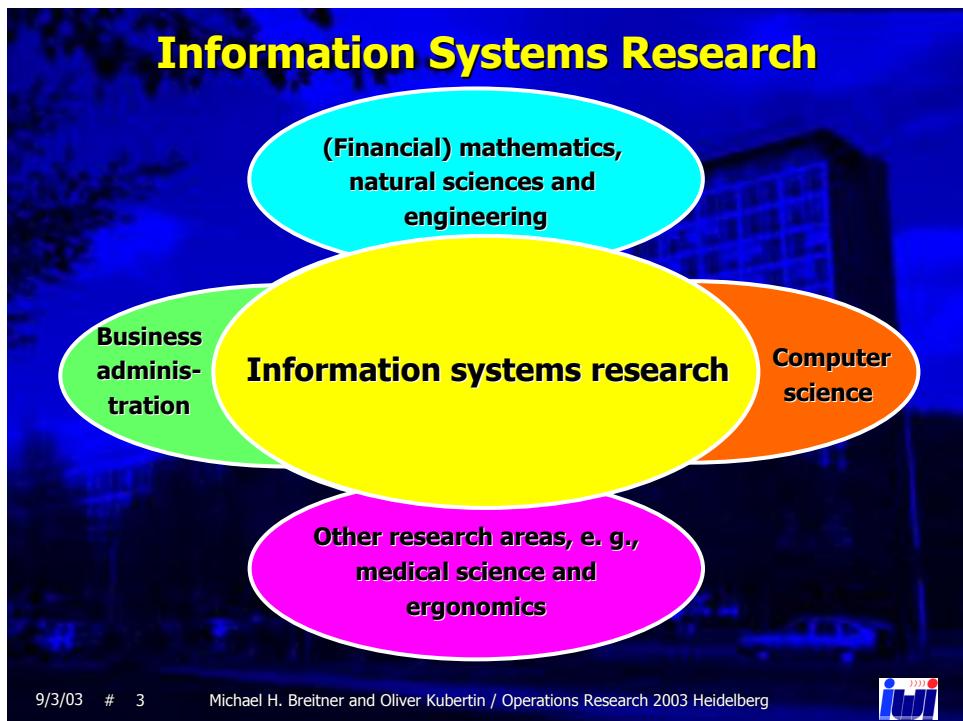
Topics

- **(Financial) Mathematics and Information Systems Research;**
- **European double-barrier options** (DAX calls);
- **Black-Scholes model** for options;
- A double-barrier DAX option **example** with a target;
- **Optimization** of European double-barrier options;
- **Software life cycle** and **software quality**;
- **WARRANT-PRO-2 Release 0.2** (2002 – 2003);
- “Perfect” European double-barrier **DAX call**;
- Other **examples**;
- **Software demonstration** (by Oliver Kubertin).

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European Double-Barrier DAX Call Options

- **Call seller** grants **call buyer** the **right** to **purchase** a "DAX-like stock portfolio" at an **a priori stipulated maturity date** (European style);
- Besides the **immediate call premium payment** the **buyer** has **no** (further) **obligations**;
- The "DAX-like stock portfolio" **price** is **a priori stipulated**, too (usually compensation instead of portfolio delivery);
- Call seller has to pay the **stipulated cash settlement** to call buyer, if the **DAX spot rate hits** the **lower barrier** before maturity (premature expiration);
- Call seller has to pay the **stipulated cash settlement** to call buyer, if the **DAX spot rate hits** the **upper barrier** before maturity (premature expiration).

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Black-Scholes Model for Options

The solution $C(t, p; r, \sigma)$ of the well known Black-Scholes-equation

$$\frac{\partial}{\partial t} C + r p \frac{\partial}{\partial p} C + \frac{1}{2} \sigma^2 p^2 \frac{\partial^2}{\partial p^2} C - r C = C_t + r p C_p + \frac{1}{2} \sigma^2 p^2 C_{pp} - r C \equiv 0, \quad (1)$$

which is a linear, homogeneous, parabolic partial differential equation of second order, yields a sufficiently accurate approximation of European DAX option values, see Hull (2003) and Redhead (1997). The time is denoted by $t \in [0, T]$, $p \in [\text{DAX}_{\min}, \text{DAX}_{\max}]$ denotes the DAX, $r > 0$ denotes the risk-free interest rate per year for the maturity period, $\sigma > 0$ denotes the implied volatility of the DAX and $T > 0$ denotes the initial maturity period at $t = 0$. Note that t and p are the independent variables in equation (1), whereas r and σ are constant parameters. The most important option greek Delta (symbol: Δ) is the first partial derivative of $C(t, p; r, \sigma)$ w.r.t. p . For an European double-barrier DAX call the boundary conditions at $p = \text{DAX}_{\min}$, $t = T$ and $p = \text{DAX}_{\max}$ correspond to the cash settlements at the option's expiration. The option expires if either one of the knock-out barriers is hit, i.e. $p = \text{DAX}_{\max}$ at the upper barrier or $p = \text{DAX}_{\min}$ at the lower barrier, or

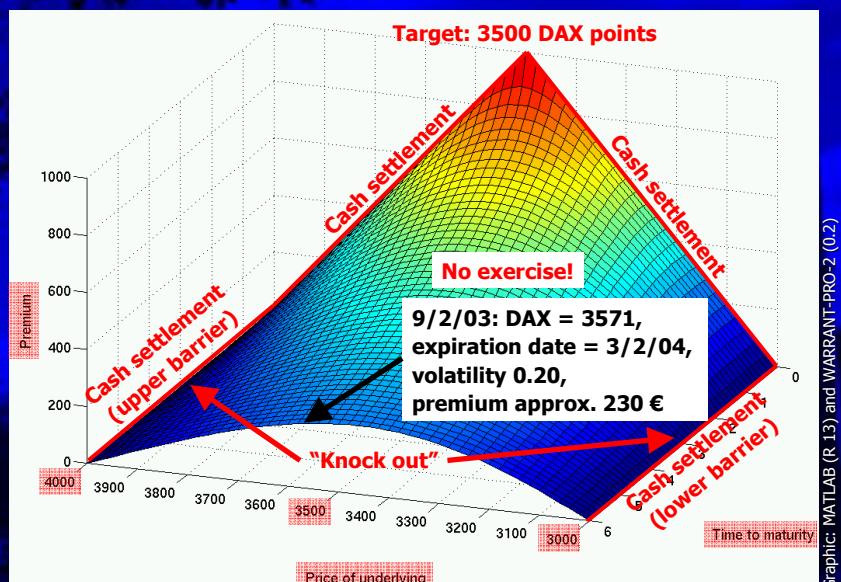
M. H. Breitner and O. Kubertin, Paper submitted for the Proceedings book of the Operations Research 2003 Heidelberg, p. 2.

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Example with a target of 3500 DAX points

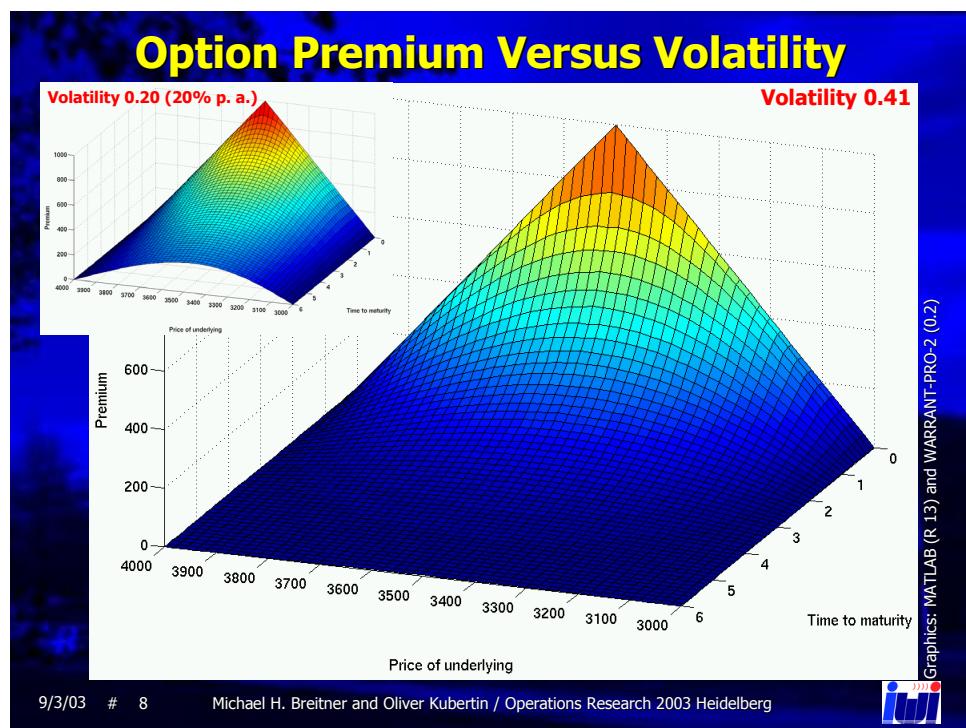
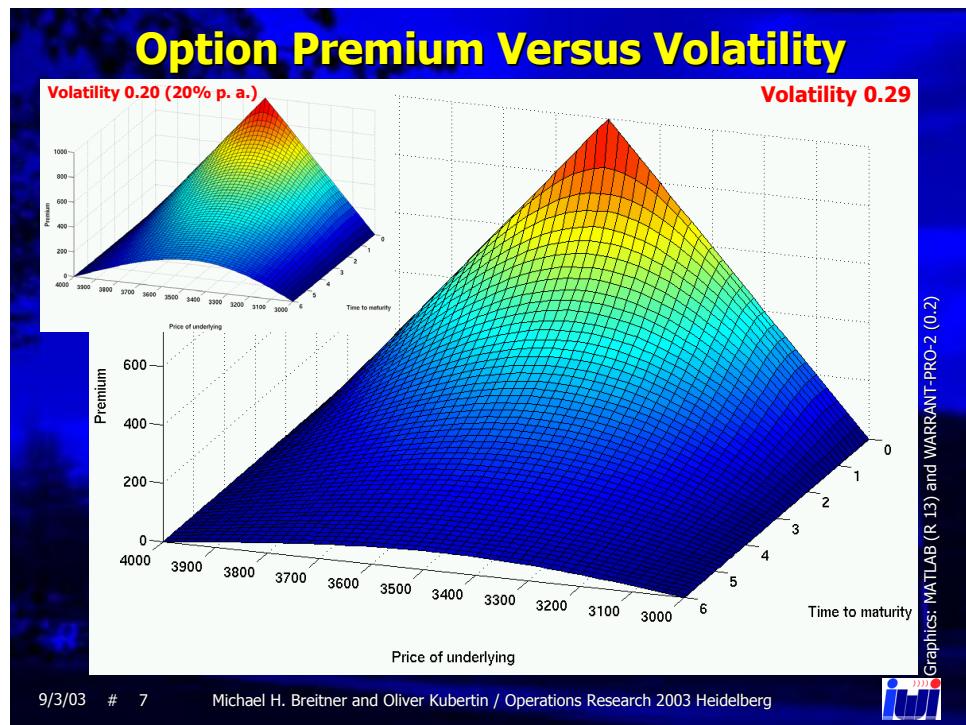


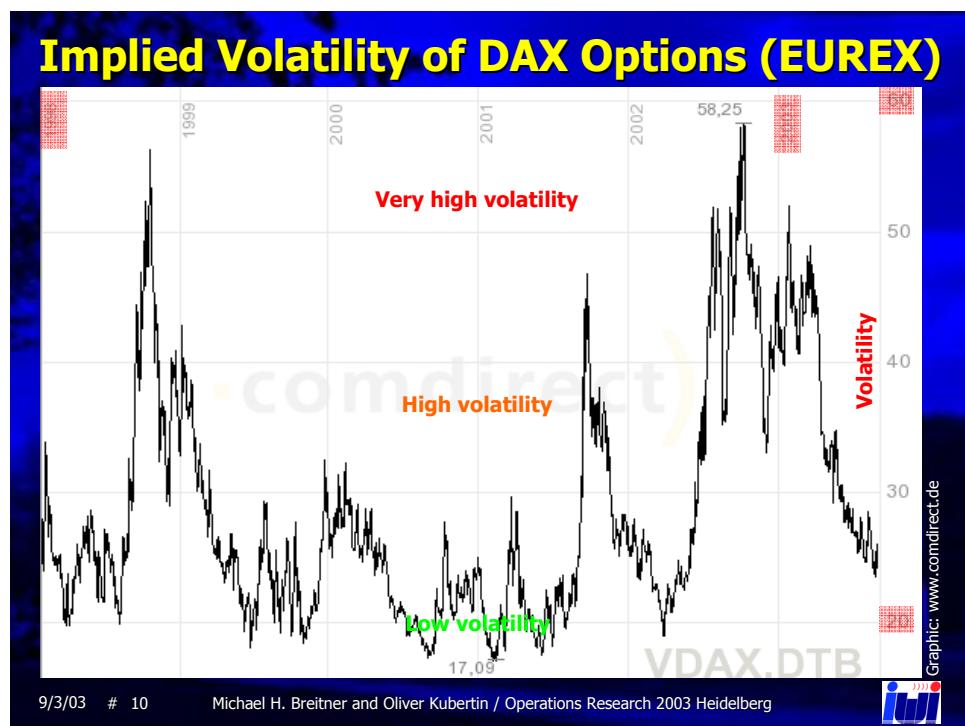
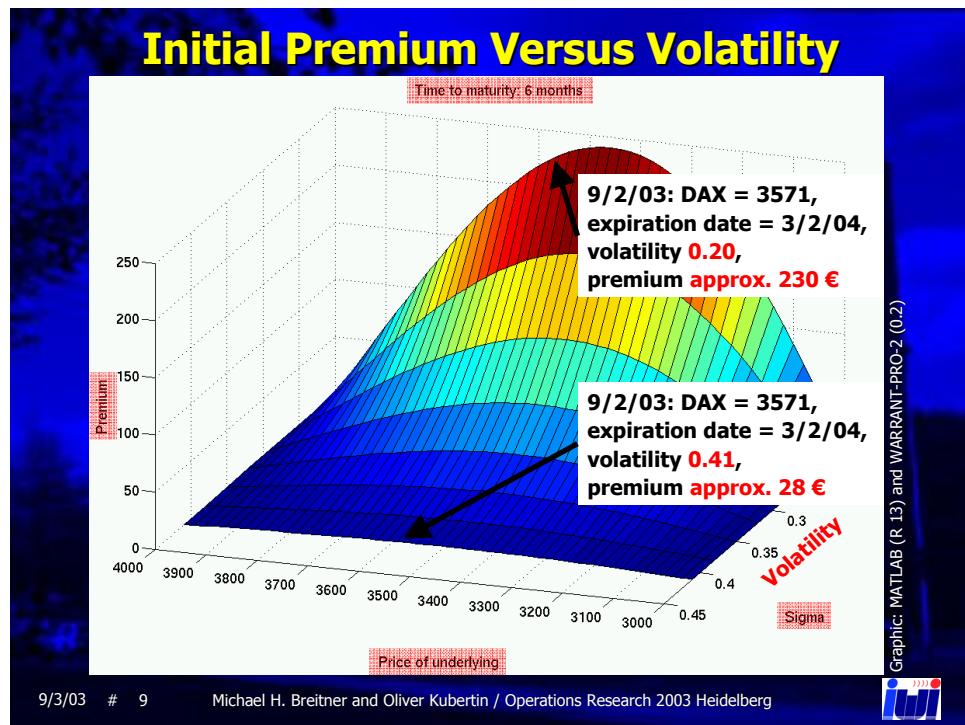
Graphic: MATLAB (R 13) and WARRANT-PRO 2 (0.2)

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Knock-Out Option without Upper Barrier

Snapshot

WKN 739659 - HEBELPRODUKT LONG AUF DAX

WKN (Basiswert): 846900	Basiswert: DAX - DEUTSCHER AKTIENINDEX (PERFORMANCEINDEX)	ISIN: DE0007396590
Typ: Long	Laufzeit: 10.12.03	Bez.-Verh.: 0,010
Strike: 2.000,00	Strike: 2.000,00	

Stammdaten		Kursdaten	
Emitent: Deutsche Bank	Währung: EUR	Kurs Basiswert: (29.08., 20.15.)	in EUR: 3.484,58
Typ: Long	Basiswert: DAX (846900)	Kurs Zertifikat: (01.09., 03.37.)	Geld in EUR: 15,25
Strike: 2.000,00 EUR	Barrier: 2.000,00 EUR	Brief in EUR: 15,28	
Barrier erreicht: Nein	Laufzeit: 10.12.03	EUWAX Realtime-Quote: (29.08., 19.59.)	
Cash/Effektiv: Cash	Börsenplätze: FRA STU	Geld in EUR: 15,12	

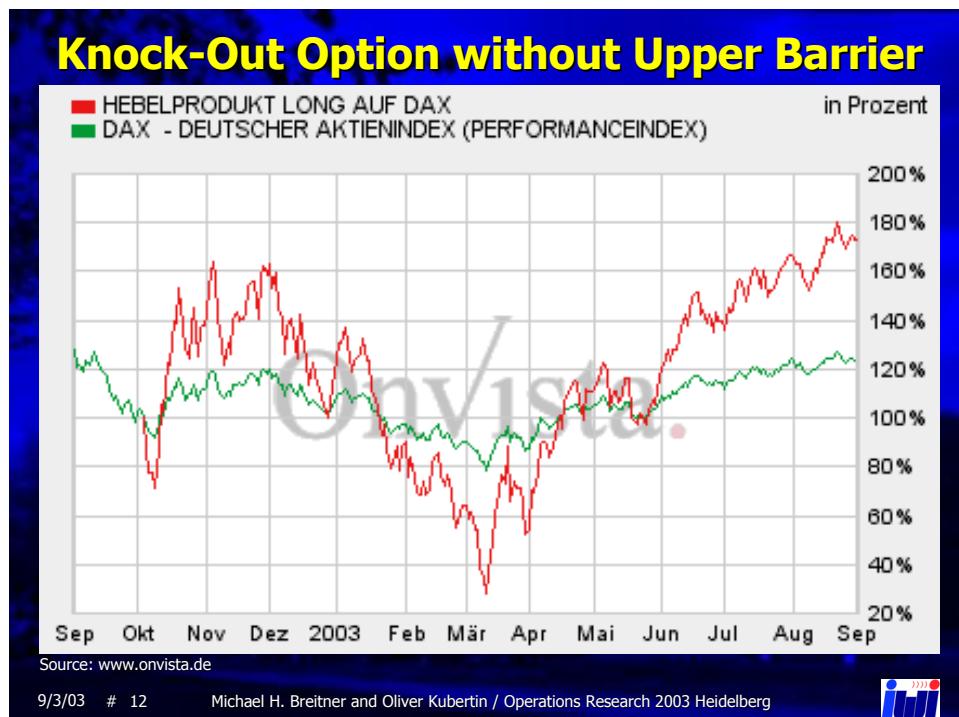
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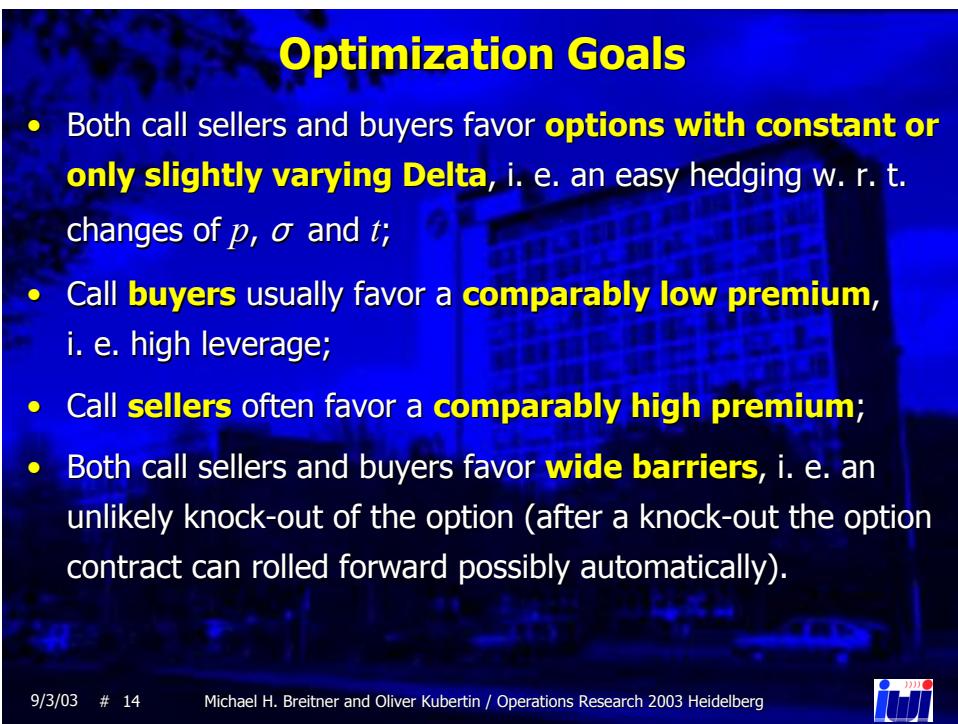
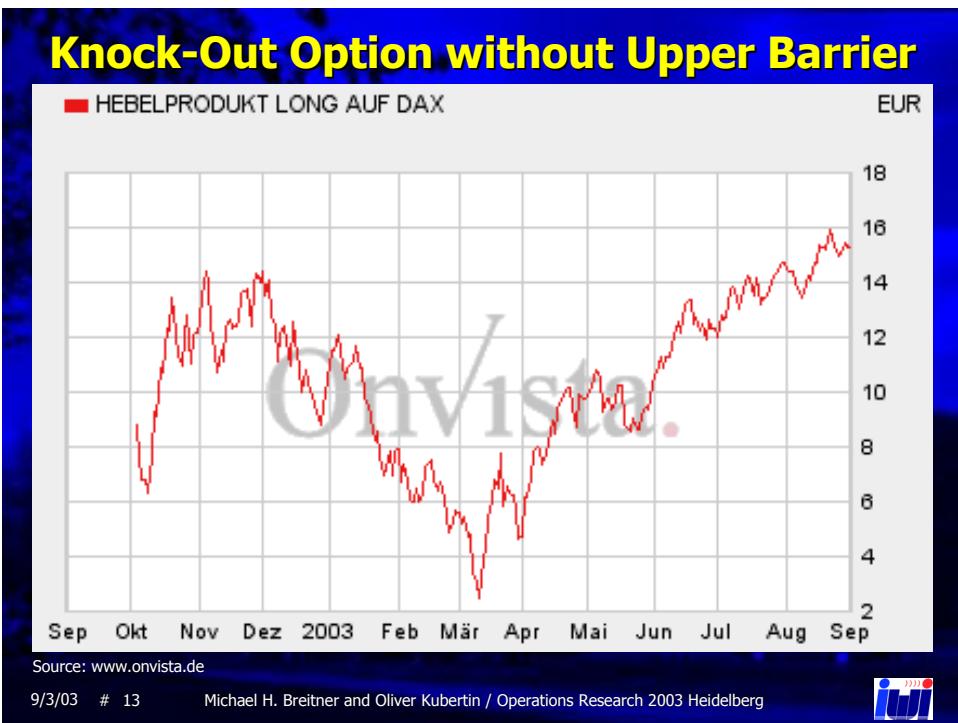
Ein Call-Optionschein mit Knock-Out ermöglicht dem Anleger, überproportional an der Entwicklung des Underlyings zu partizipieren. Die dabei entstehende Hebelwirkung ist umso größer, je näher die Barrier am Ausgangskurs des Underlyings liegt und je geringer der Kapitaleinsatz damit wird. Falls das Underlying allerdings während der Laufzeit zu irgendeinem Zeitpunkt (auch intraday) die Barrier berührt oder unterschreitet, verfällt der Schein wertlos. Ansonsten erfolgt eine Rückzahlung in Höhe von (Guthaben - Basiswert) multipliziert mit dem Hebeleffektivitätsfaktor.

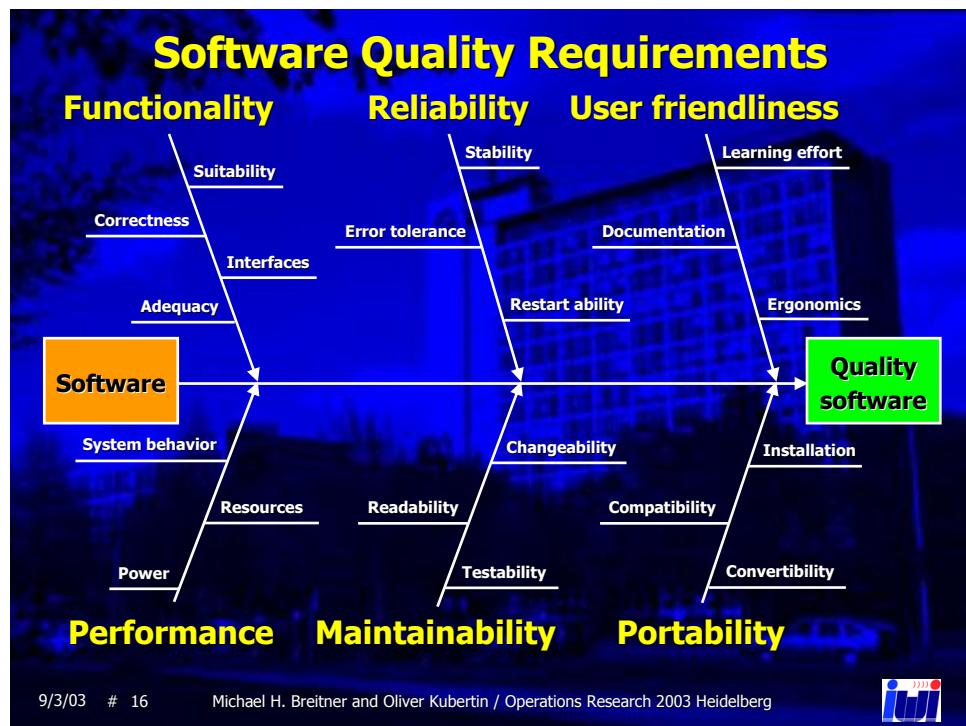
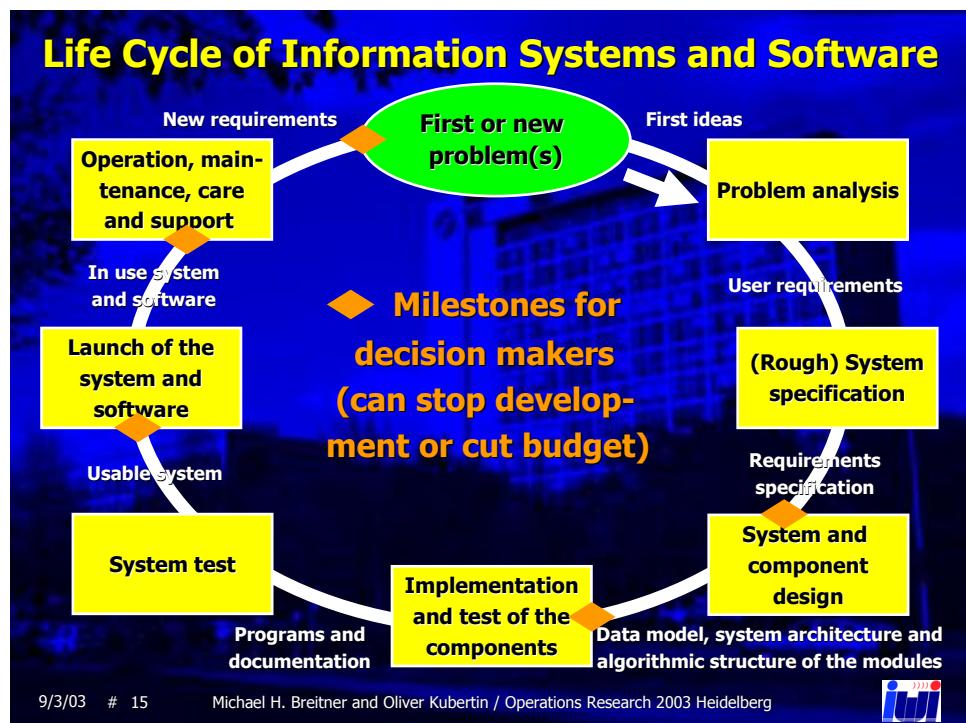
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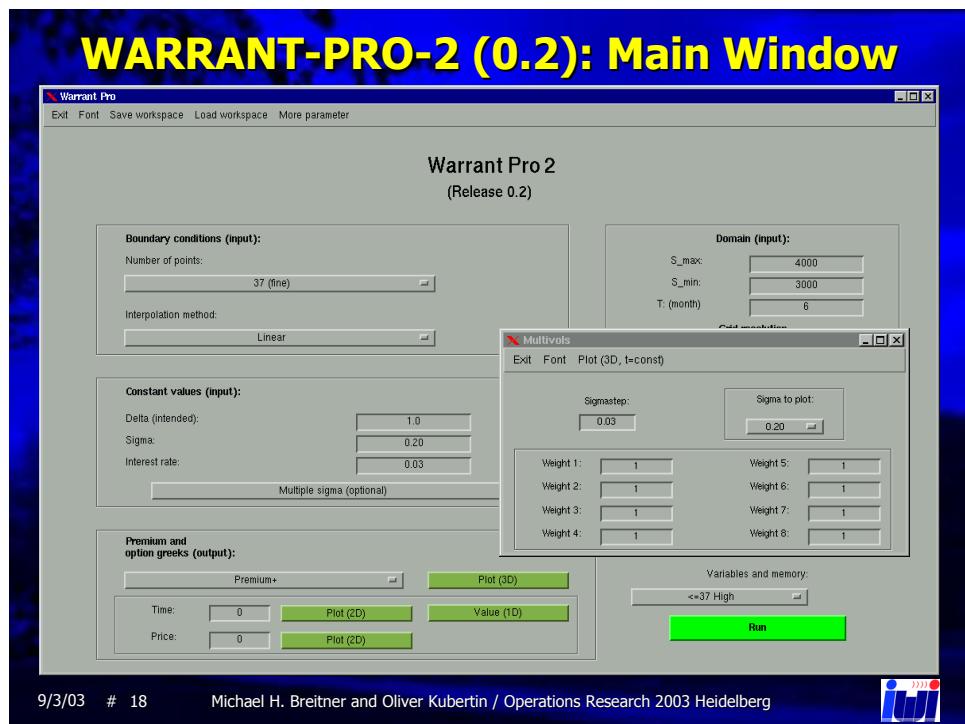
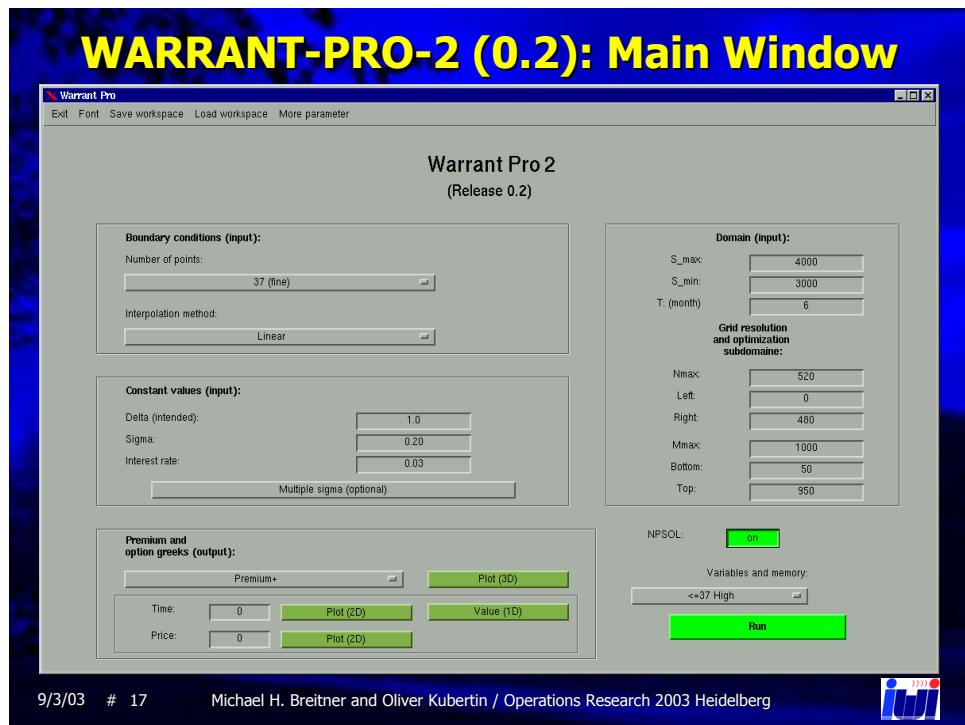
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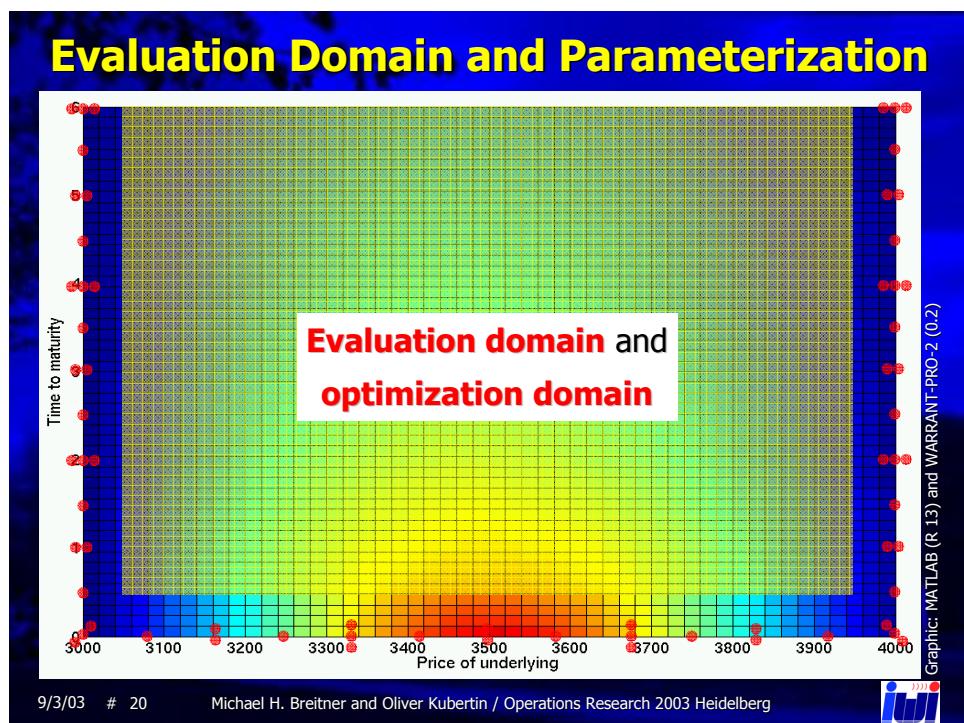
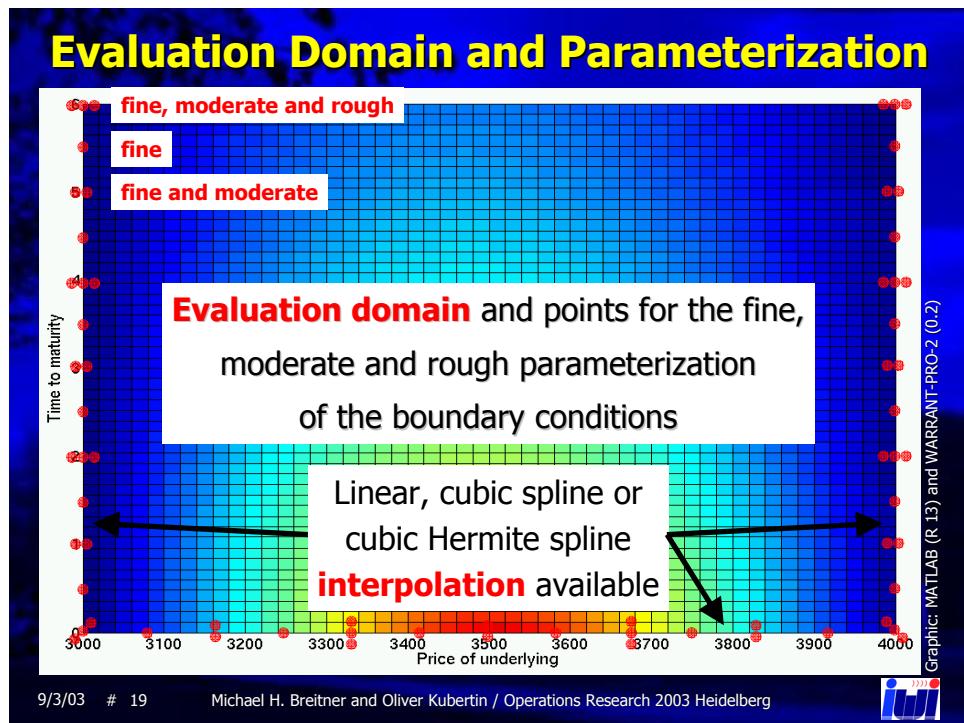


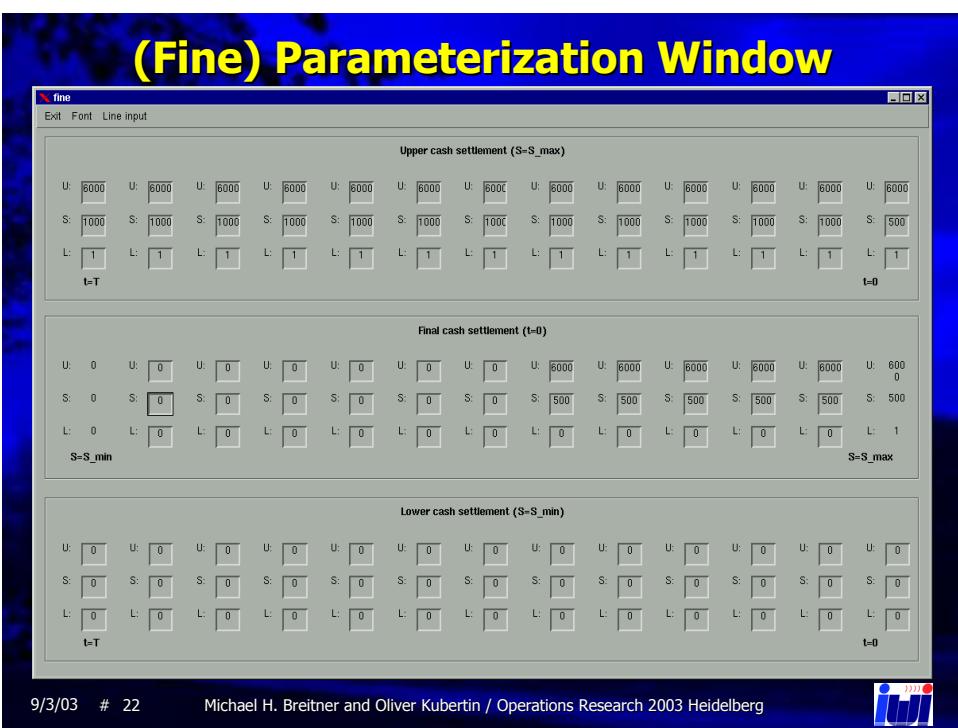
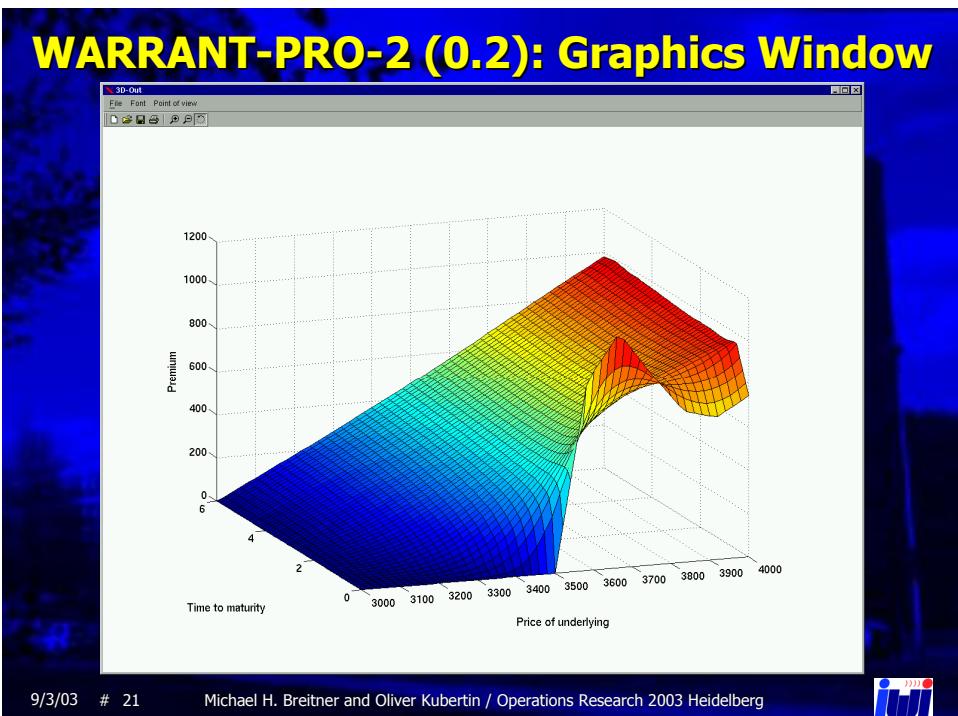












Overview WARRANT-PRO-2 Highlights

- User and programmer **documentation** and various ready to use **examples** (R 0.1 and 0.2);
- **Portable ANSI FORTRAN 77 kernel** (R 0.1 and 0.2);
- Input and output **files interface** concept (R 0.1 and 0.2);
- An option's **deviation** from a **predefinable Delta** (> 0 or < 0) is the **performance index** and is minimized (R 0.1 and 0.2);
- Boundary conditions' parameters are optimized by **nonlinear programming**, i. e. the **advanced sequential quadratic programming (SQP) method** NPSOL developed by P. E. Gill et al. (R 0.1 and 0.2);

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Overview WARRANT-PRO-2 Highlights

- **Terminal control** without GUI (graphical user interface), e. g. on LINUX, UNIX or WINDOWS compute servers (R 0.1 and 0.2);
- **User friendly, stand alone GUI** for LINUX (R 0.2, WINDOWS in preparation);
- **High quality MATLAB graphics** (R 0.2);
- **Graphics** for the most important **option greeks** (R 0.1 and 0.2);

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Overview WARRANT-PRO-2 Highlights

- Black-Scholes equation is solved with a **very fast** and **very accurate** numerical **Crank-Nicholson scheme** (finite difference implicit/explicit method) with **quadratic global error convergence** with space and time discretization refinement (R 0.2);
- The **gradient** of the performance index is computed highly accurate and fast with **automatic/algorithmic differentiation** using ADIFOR 2.0 developed by M. W. Fagan et al. (R 0.2);
- **RAM allocation adoption** (R 0.2);
- “Small problems” need large level-2 or level-3 caches (4 MB – 8 MB) only (R 0.2);

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Overview WARRANT-PRO-2 Highlights

- **All cash settlements**, i. e. boundary conditions of the Black-Scholes equation, are **optimizable** (R 0.2);
- **Very robust** and **very fast convergence**, i. e. P4 2 GHz computing times < 15 seconds and < 5 minutes for simulation and optimization, resp., for all examples (R 0.2);
- **Rapid evaluation/simulation** and **design/optimization** of (almost) all **barrier** and **plain vanilla options** (R 0.2);
- **Call** type, **put** type and **mixed type options** (R 0.2).

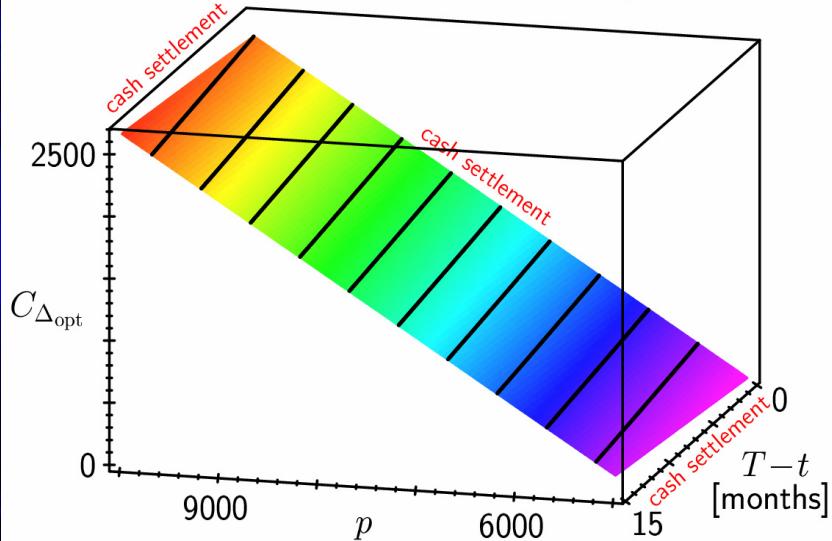
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DAX Call with Constant Delta (Strike 5000)

$\text{DAX}_{\min} = 5000, \text{DAX}_{\max} = 10000, \Delta_{\text{opt}} = 0.5$



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Graphic: Breitner, M. H., and Burmester, T. (2002): Optimization of European Double-Barrier Options via Optimal Control of the Black-Scholes-Equation, in Chamoni, P., et al. (Eds.), Operations Research '01, 167 – 174, Springer, Berlin



DAX Call with Constant Delta (Strike 5000)

$(t, p) \in [0, T] \times [\text{DAX}_{\min}, \text{DAX}_{\max}]$ we get $C_{pp} \equiv 0$. Equation (1) simplifies to a partial differential equation of first order with the solution

$$C_{\Delta_{\text{opt}}} (t, p; r) = \Delta_{\text{opt}} \left(p - \text{DAX}_{\min} e^{-r(T-t)} \right), \quad (2)$$

which doesn't depend on σ , i. e. the anticipated future DAX volatility has no influence on the DAX call value. Solution (2) is unique solution of equation (1) if the initial/boundary conditions are adjusted to the analytic solution

$$C_{\Delta_{\text{opt}}} (t=T, p; r) = \Delta_{\text{opt}} (p - \text{DAX}_{\min}) \text{ for } p \in [\text{DAX}_{\min}, \text{DAX}_{\max}], \quad (3)$$

$$C_{\Delta_{\text{opt}}} (t, p=\text{DAX}_{\min}; r) = \Delta_{\text{opt}} \text{DAX}_{\min} \left(1 - e^{-r(T-t)} \right) \text{ for } t \in [0, T[\text{ and } \quad (4)$$

$$C_{\Delta_{\text{opt}}} (t, p=\text{DAX}_{\max}; r) = \Delta_{\text{opt}} \left(\text{DAX}_{\max} - \text{DAX}_{\min} e^{-r(T-t)} \right) \text{ for } t \in [0, T[. \quad (5)$$

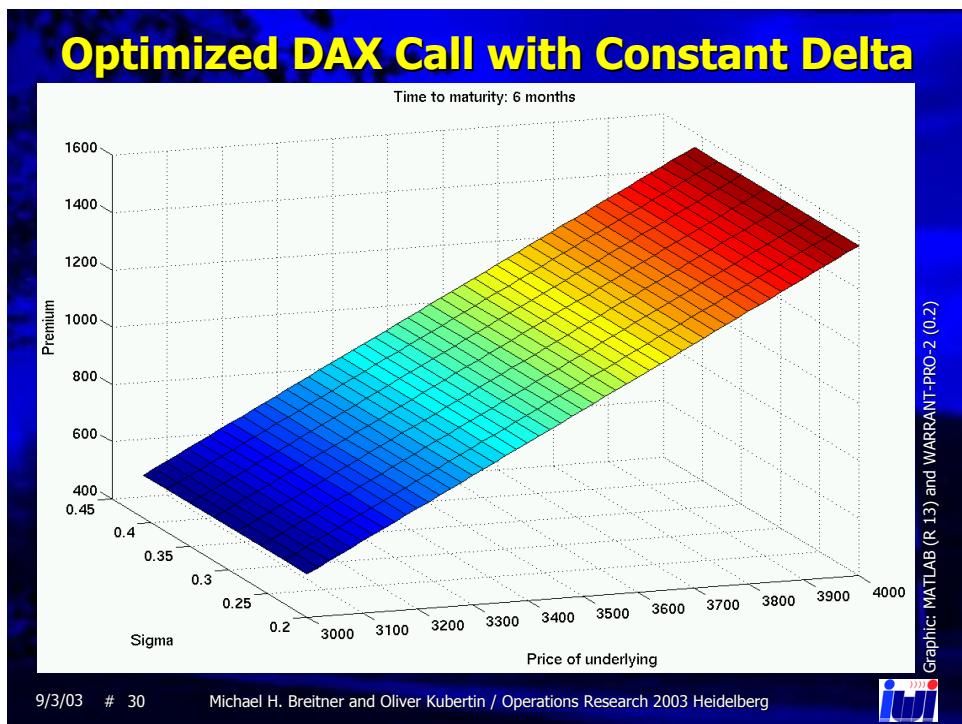
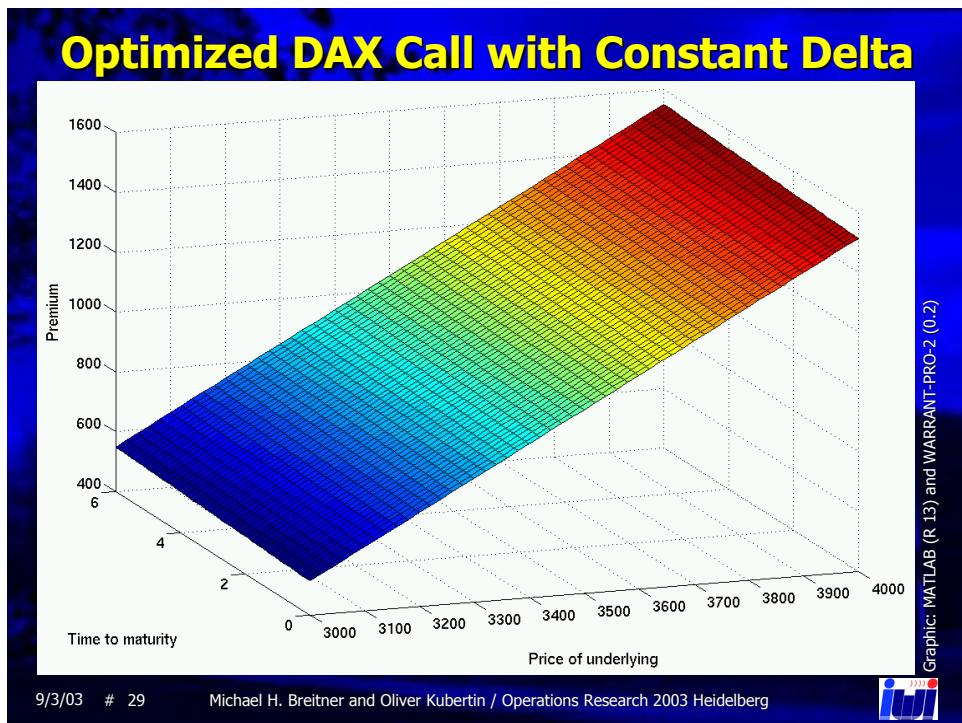
Dealing with an European double-barrier DAX call these initial/boundary conditions correspond to the cash settlements at the option's expiration. The

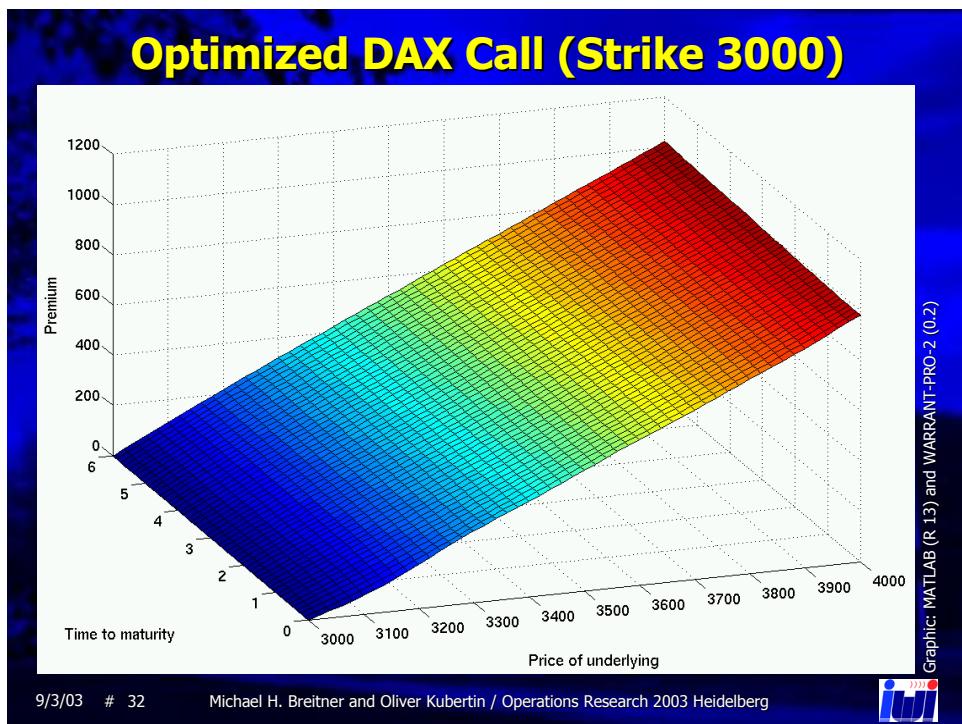
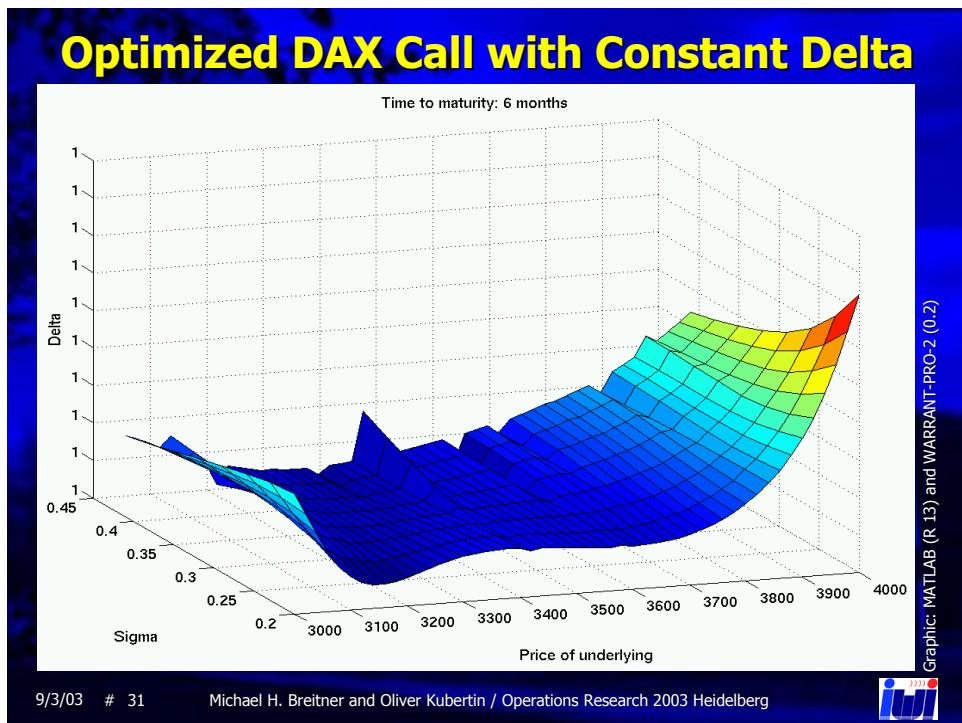
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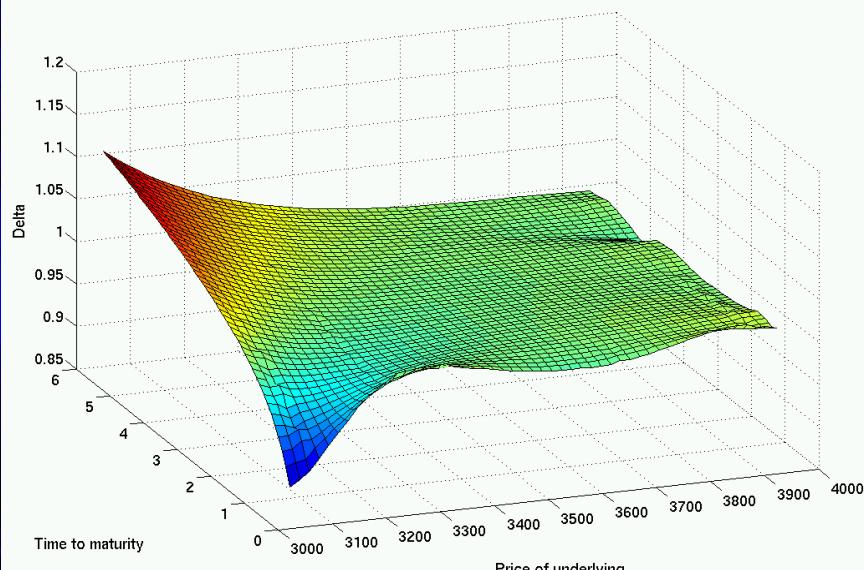
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Optimized DAX Call (Strike 3000)



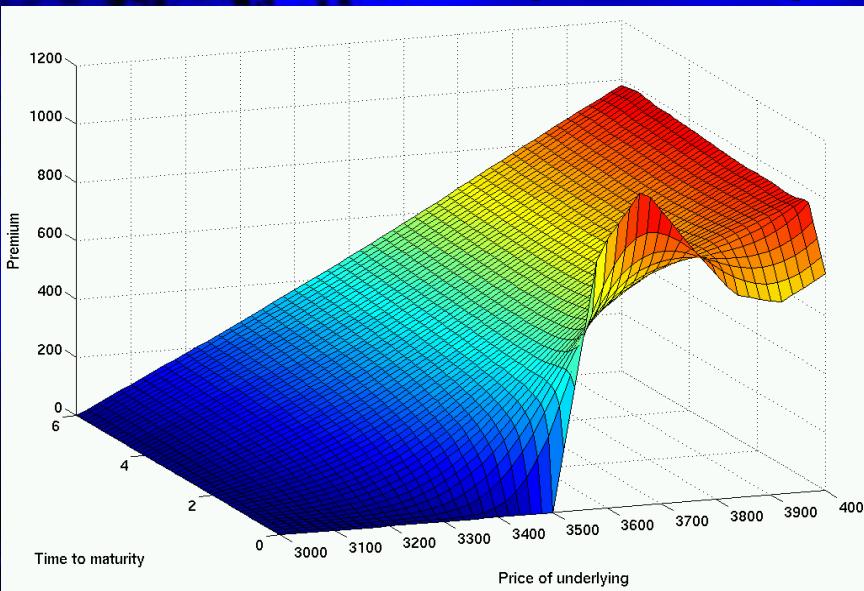
Graphic: MATLAB (R 13) and WARRANT-PRO 2 (0.2)

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Optimized DAX Call (Strike 3500)

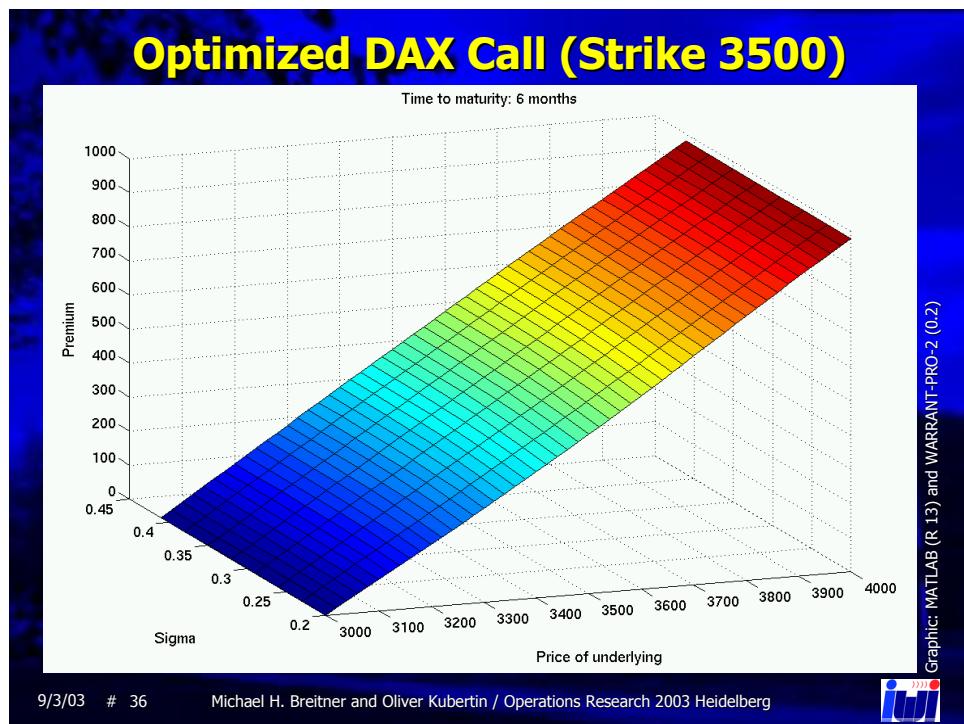
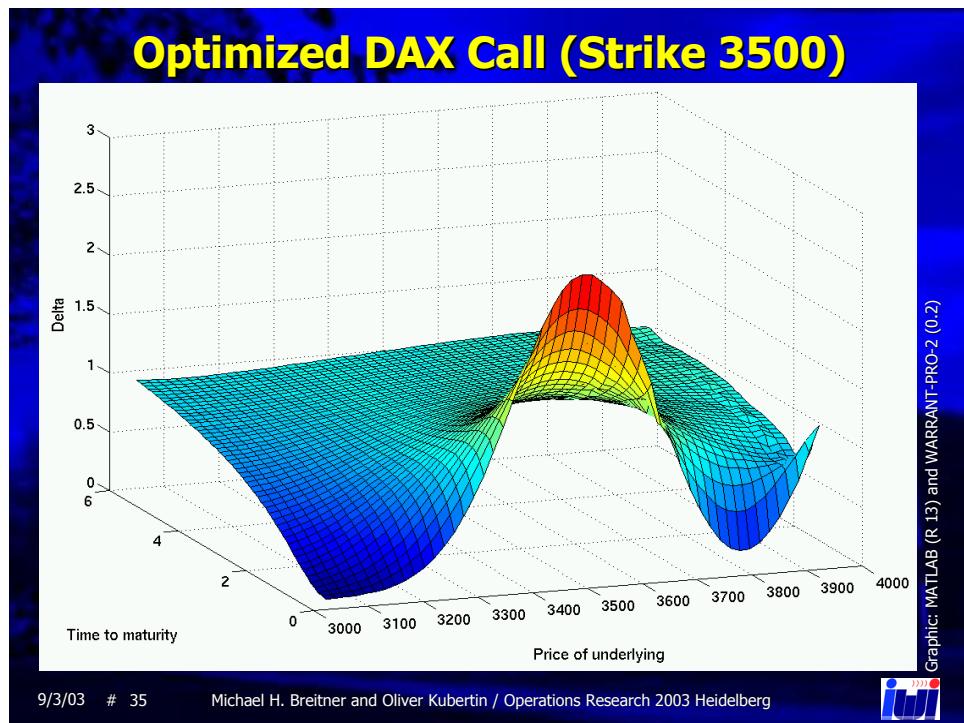


Graphic: MATLAB (R 13) and WARRANT-PRO 2 (0.2)

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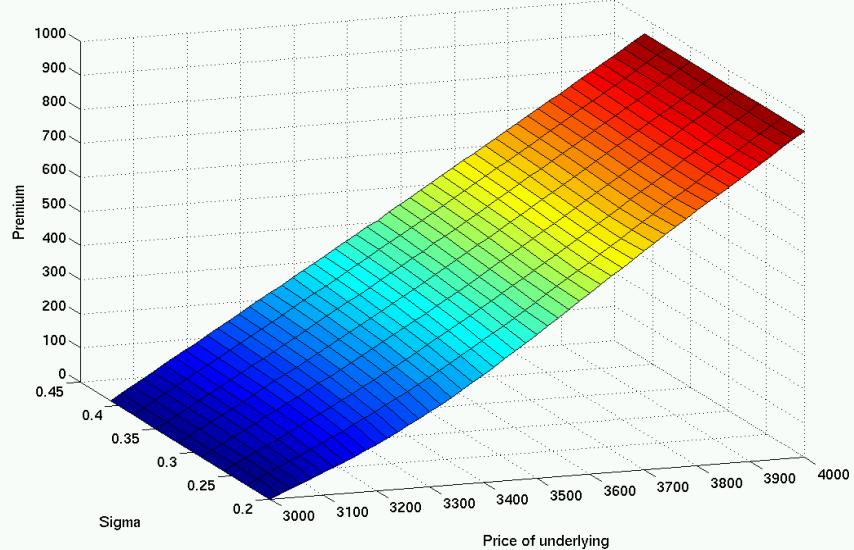
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Optimized DAX Call (Strike 3500)

Time to maturity: 3 months



Graphic: MATLAB (R 13) and WARRANT-PRO 2 (0.2)

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**P. Chamoni · R. Leisten · A. Martin
J. Minnemann · H. Stadtler**
Editors

Operations Research Proceedings 2001

Selected Papers
of the International Conference
on Operations Research (OR 2001)

Duisburg, September 3–5, 2001

With 88 Figures
and 38 Tables

ISBN 3-540-43344-9 Springer-Verlag Berlin Heidelberg New York

Library of Congress Cataloging-in-Publication Data applied for
Die Deutsche Bibliothek – CIP-Einheitsaufnahme
International Conference on Operations Research. 2001. Duisburg:
Selected Papers of the International Conference on Operations Research:
Duisburg, September 3–5, 2001; with 38 Tables (OR 2001). P. Chamoni ... (ed.).—
Berlin; Heidelberg; New York; Barcelona; Hong Kong; London; Milan; Paris; Tokyo:
Springer, 2002
(Operations Research Proceedings; 2001)
ISBN 3-540-43344-9

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Cover design: Erich Kirchner, Heidelberg

SPIN 10872140 4 3 2 1 0 - Printed on acid-free paper



Preface

This volume contains the proceedings of the 2001 International Conference on Operations Research (OR 2001) held at the Gerhard-Mercator-University Duisburg, September 3-5, 2001.

OR 2001 was organized under the auspices of the German Society of Operations Research, Gesellschaft für Operations Research (GOR e. V.).

The conference and the annual general meeting were attended by 360 participants from 20 countries. The presentation of 220 papers was organized in 15 sections.

According to Duisburg as hosting city for this event OR 2001 emphasized on contributions of OR in the areas of energy, transport and traffic.

The program consisted of 2 plenary lectures (Reinhard Selten and Jörg Hennerkes) and 15 invited semiplenary lectures. 97 papers were submitted for publication. Following the advice of the section chairs the program committee decided to accept 59 papers for this volume.

The selected manuscripts will be published also in electronic form on the World Wide Web at

<http://www.uni-duisburg.de/or2001>.

We want to thank all referees and authors for delivering their final manuscript in due time. We are also grateful to the other members of the local organizing committee and especially to Stefan Krebs, Corinna Schu and David Betge for the perfect conference management. Roland Düsling, Ralph Gollmer and Steffen Stock supported us in editing the abstracts and the final version of this proceeding volume. Last but not least thanks to all the assistants and student assistants for their operations on OR 2001 in Duisburg.

Duisburg, December 2001

Peter Chamoni and Rainer Leisten (editors)

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Optimization of European Double-Barrier Options via Optimal Control of the Black-Scholes-Equation

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Abstract. In comparison to future contracts today's options have advantages and disadvantages. DAX-future contracts, e.g. the EUREX FDAX, have a constant profit/loss per DAX point, whereas the profit/loss per DAX point of common options varies undesirably with the DAX. An advantage of buying options results from the immediate payment of the option premium. There are no further payments during the life of the option. Similarly margin requirements force a FDAX buyer/seller to pay an initial margin plus a safety margin immediately. But, in the event of adverse DAX movements the buyer/seller has to meet calls for substantial additional margin and may be forced to liquidate his position prematurely. Optimized European double-barrier options can combine the advantages of futures and options. The option expires if the DAX either hits the upper knock-out barrier DAX_{\max} or the lower knock-out barrier DAX_{\min} , or if the option has matured. We present two approaches to optimize the cash settlements at expiration either analytically or numerically.

Keywords. Financial derivatives, options, futures, hedging tactics, Black-Scholes-model, optimal control, optimization, partial differential equation.

1 Real Life Case Study

A large German insurance company has a DAX-like stock portfolio worth 1 billion Euro ($= 10^9$ Euro). The DAX is 6000 points. "DAX" is used for the XETRA spot rate of the German stock index DAX 30. The company predicts decreasing stock values and wishes to immunize the portfolio, i.e. to generate a risk-free portfolio. Tax issues, market conditions or regulations inhibit the company from selling stocks.

2 Hedging with DAX Futures

The insurance company has the option to sell DAX futures at the EUREX, i.e. to open FDAX-short positions. One FDAX-short position has a profit/loss

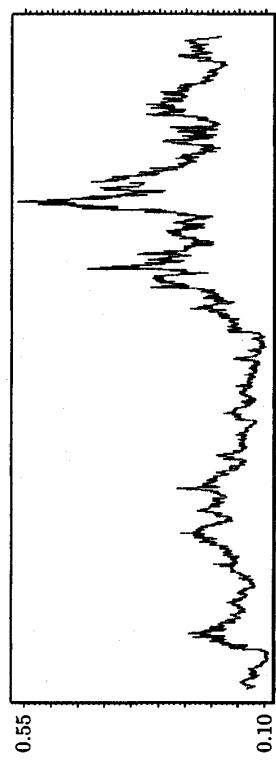


Fig. 1. Implied three-months at the money volatility (EUREX VDAX) of EURONEXT options on the DAX. Source: Market-Maker (Release 1.40).

Δ_{FDAX} per DAX point of 25 Euro. Thus the company has to sell $10^9/(25 * 6000) \approx 6667$ DAX futures to immunize the stock portfolio. The EUREX clearing agency demands an initial margin plus a safety margin immediately. The total margin per FDAX-short varies with the implied DAX volatility (EUREX VDAX) and is, e.g., about 5000 Euro for a low volatility of 0.17 and about 20000 Euro for a high volatility of 0.45. The total margin is between 3.33 % and 13.33 % of the stock portfolio value. In the event of an increasing DAX the company has to meet calls for additional margin, i.e. 166675 Euro per DAX point, and may be forced to liquidate its position prematurely. Moreover, the permanent margin adaption at the EUREX clearing agency is quite troublesome. Another disadvantage is the margin variation dependent on the VDAX, which oscillated between 0.10 and 0.55 during the last decade, see Fig. 1.

3 Hedging with DAX Calls with Constant Δ

The solution $C(t, p; r, \sigma)$ of the well known Black-Scholes-equation

$$\frac{\partial}{\partial t} C + rp \frac{\partial}{\partial p} C + \frac{1}{2} \sigma^2 p^2 \frac{\partial^2}{\partial p^2} C - r C = C_t + rpC_p + \frac{1}{2} \sigma^2 p^2 C_{pp} - rC \equiv 0, \quad (1)$$

which is a linear, homogeneous, parabolic partial differential equation of second order, yields a sufficiently accurate approximation of European DAX option values, see Hull (2000) and Redhead (1997). The time is denoted by $t \in [0, T]$, $p \in [DAX_{\min}, DAX_{\max}]$ denotes the DAX, $r > 0$ denotes the risk-free interest rate per year for the maturity period, $\sigma > 0$ denotes the implied volatility of the DAX and $T > 0$ denotes the initial maturity period at $t = 0$. Note that t and p are the independent variables in the linear, homogeneous, parabolic partial differential equation (1) of second order, whereas r and σ are constant parameters. For the profit/loss Δ per DAX point there holds $\Delta = C_p$. Assuming $\Delta \equiv \Delta_{\text{opt}}$, $\Delta_{\text{opt}} \geq 0$, for all

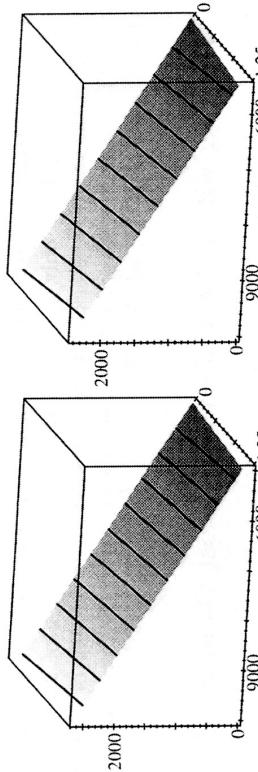


Fig. 2. Black-Scholes-values of optimal European double-barrier DAX calls with $\Delta_{\text{optabc}} \equiv 0.5$ for all DAX in $[5000, 10000]$ (left, strike $DAX_{\min} = 5000$) and in $[5500, 10000]$ (right, strike $DAX_{\min} = 5500$) and for all maturity periods of $[0, 1.25]$ years. Contour lines represent values of 250 Euro, 500 Euro, ...
 $(t, p) \in [0, T] \times [DAX_{\min}, DAX_{\max}]$ we get $C_{pp} \equiv 0$. Equation (1) simplifies to a partial differential equation of first order with the solution

$$C_{\Delta_{\text{opt}}} (t, p; r) = \Delta_{\text{opt}} \left(p - DAX_{\min} e^{-r(T-t)} \right), \quad (2)$$

which doesn't depend on σ , i.e. the anticipated future DAX volatility has no influence on the DAX call value. Solution (2) is unique solution of equation (1) if the initial/boundary conditions are adjusted to the analytic solution

$$C_{\Delta_{\text{opt}}} (t = T, p, r) = \Delta_{\text{opt}} (p - DAX_{\min}) \quad \text{for } p \in [DAX_{\min}, DAX_{\max}], \quad (3)$$

$$C_{\Delta_{\text{opt}}} (t, p = DAX_{\min}; r) = \Delta_{\text{opt}} DAX_{\min} \left(1 - e^{-r(T-t)} \right) \quad \text{for } t \in [0, T] \quad \text{and} \quad (4)$$

$$C_{\Delta_{\text{opt}}} (t, p = DAX_{\max}; r) = \Delta_{\text{opt}} (DAX_{\max} - DAX_{\min} e^{-r(T-t)}) \quad \text{for } t \in [0, T]. \quad (5)$$

Dealing with an *European double-barrier DAX* call these initial/boundary conditions correspond to the cash settlements at the option's expiration. The option expires if either one of the knock-out barriers is hit, i.e. $p = DAX_{\max}$ at the upper barrier or $p = DAX_{\min}$ at the lower barrier, or if the expiration date is reached, i.e. $t = T$, see Fig. 2. Considering the case study the insurance company can sell $10^9/(0.5 * 6000) \approx 333333$ optimal DAX calls with $\Delta \equiv 0.5$ (covered call writing). The company accepts the obligation to pay the cash settlement (3), (4) or (5) at the option's expiration. The company's premium per written call at $t = 0$ is 651.47 Euro for a strike $DAX_{\min} = 5000$ and 416.61 Euro for a strike $DAX_{\min} = 5500$ if there holds $p = 6000$, $r = 0.05$ and $T = 1.25$ years, see Fig. 2. There are no further payments during the life of the option. The buyer of 50 options can easily synthesize 1 FDAX-long position costing 32573.50 Euro or 20830.50 Euro, respectively. Selling 50 options the insurance company easily synthesizes 1 FDAX-short position. In comparison a plain vanilla European DAX call has a profit/loss $\Delta_{\text{pvEc}}(t, p; r, \sigma)$ per DAX point, which strongly depends on p and σ and moderately on t and r , see

Fig. 3 for a strike $s = 7000$. The company has to pay the cash settlement $\max(p(T) - s, 0)$ Euro only at $t = T$. Due to the variation of Δ_{pvEc} the hedging of the DAX-like stock portfolio is tedious. The number of options required for a risk-free portfolio changes permanently with the German stock market conditions (DAX and VDAX).

4 Hedging with DAX Calls with Almost Constant Δ

Today European double-barrier options are well accepted at the financial markets. However, the non-constant cash settlements (4) and (5) at the knock-out barriers are quite extraordinary. Usually the payments at the barriers are constant, i.e.

$$C(t, p = DAX_{\min}; r, \sigma) \equiv 0 \quad \text{and} \quad C(t, p = DAX_{\max}; r, \sigma) \equiv C_{\text{up}} \quad \text{with} \quad C_{\text{up}} > 0. \quad (6)$$

In contrast a more complicate payment $C(t = T, p; r, \sigma) = C_T(p)$ is acceptable. Thus C_{up} and $C_T(p)$, $p \in [DAX_{\min}, DAX_{\max}]$, are optimized to get an almost constant, predefined profit/loss $\Delta_{\text{abc}}(t, p; r, \sigma) \approx \Delta_{\text{opt}}$ per DAX point for all relevant p , σ and t , see Hoffmann (1999) for an overview of optimal control problems governed by partial differential equations.

In order to optimize $\Delta_{\text{abc}}(t, p; r, \sigma)$ the Black-Scholes-equation (1) has to be solved numerically. Here the Crank-Nicholson method is used, which is a combination of an explicit and an implicit finite difference method, see Ames (1992), Hull (2000), Prismann (2000), Seydel (2000) and Thomas (1998). With a time discretization Δt and DAX discretization Δp the Crank-Nicholson method converges robustly, i.e. the absolute error of the numerical solution decreases by $\mathcal{O}(\Delta t^2) + \mathcal{O}(\Delta p^2)$ for $\max(\Delta t, \Delta p) \rightarrow 0$. The computing time increases by $\mathcal{O}(1/(\Delta t * \Delta p))$ for $\max(\Delta t, \Delta p) \rightarrow 0$. We consider options with $t \in [0, 1.25]$ years, $p \in [4000, 10000]$, $T = 1.25$ years, $r = 0.05$, $s = 7000$ and $\sigma \in [0.17, 0.45]$. The parameterized initial/boundary conditions are

$$C_T(p) = \begin{cases} 0 & \text{for } p \in [4000, 7000], \\ \begin{cases} x_1(p - 7000)/600 & \text{for } p \in [7000, 7600], \\ (x_1(8200 - p) + x_2(p - 7600))/600 & \text{for } p \in [7600, 8200], \\ (x_2(8800 - p) + x_3(p - 8200))/600 & \text{for } p \in [8200, 8800], \\ (x_3(9400 - p) + x_4(p - 8800))/600 & \text{for } p \in [8800, 9400], \\ (x_4(10000 - p) + x_5(p - 9400))/600 & \text{for } p \in [9400, 10000], \end{cases} & \text{otherwise} \end{cases} \quad (7)$$

$$C(t, p = 4000; r, \sigma) = 0 \quad \text{for } t \in [0, 1.25[\quad \text{and} \quad (8)$$

$$C(t, p = 10000; r, \sigma) = x_5 \quad \text{for } t \in [0, 1.25[, \quad (9)$$

where $C_T(p) = C(t = 1.25, p; r, \sigma)$ and $\mathbf{x} = (x_1, x_2, \dots, x_5)^T$ is the vector of parameters to optimize with $0 \leq x_i \leq 3000$ for all i . We consider the relevant implied DAX volatilities $\sigma_j = 0.17 + 0.04(j - 1)$ for $j = 1, 2, \dots, 8$ and compute approximations $CCN(t, p; 0.05, \sigma_j, \mathbf{x})$ on a 1200×1200 grid, i.e.

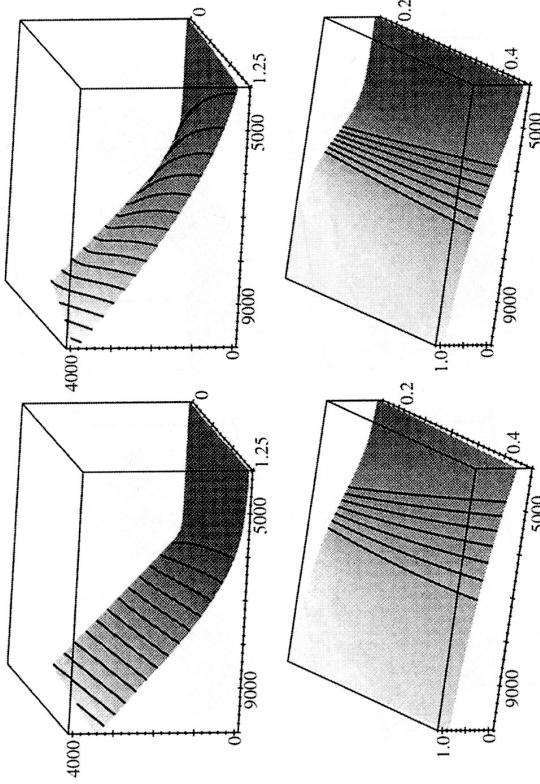


Fig. 3. Black-Scholes-value of a plain vanilla European DAX call dependent on the DAX in [4000, 10000] and the maturity period in [0, 1.25] years for an implied DAX volatility of 0.17 (upper left) and 0.45 (upper right) and Δ_{pvEc} of this call dependent on the DAX and the implied DAX volatility in [0.17, 0.45] for a maturity period of 1.25 years (lower left) and 0.417 years (lower right). Contour lines represent $\Delta_{pvEc} \equiv 0.35$, $\Delta_{pvEc} \equiv 0.40$, \dots , $\Delta_{pvEc} \equiv 0.65$.

with $\Delta t = 1.25/1200$ and $\Delta p = (10000 - 4000)/1200$. With $C_{j,k,l}(\mathbf{x}) := C_{CN}(k \Delta t, 4000 + l \Delta p; 0.05, \sigma_j, \mathbf{x})$, $w_1 = w_2 = w_3 = 5$, $w_4 = w_5 = 2$ and $w_6 = w_7 = w_8 = 1$ we minimize the Euclidian norm error function

$$\varepsilon(\mathbf{x}) := \sum_{j=1}^8 w_j \sum_{k=0}^{960} \sum_{l=100}^{1100} \left(\Delta_{opt} - \frac{C_{j,k,l+1}(\mathbf{x}) - C_{j,k,l-1}(\mathbf{x})}{2 \Delta p} \right)^2 \quad (10)$$

w. r. t. \mathbf{x} . Only the relevant $(t, p) \in [0, 1.0] \times [4500, 9500]$ are considered and the more important volatilities have larger weights w_j . The profit/loss Δ_{aoEdbc} per DAX point of the simplified optimizable European double-barrier DAX call is computed with the symmetric difference approximation $(C_{j,k,l+1}(\mathbf{x}) - C_{j,k,l-1}(\mathbf{x}))/(2 \Delta p)$ for the first partial derivative of $C(k \Delta t, 4000 + l \Delta p; 0.05, \sigma_j, \mathbf{x})$ w. r. t. p . The error function $\varepsilon(\mathbf{x})$ is minimized with the sequential quadratic programming (SQP) method NPSOL, see Gill et al. (2000). The total computing time for a Pentium III/1 GHz PC is less than 5 minutes. Various computations have shown that $\varepsilon(\mathbf{x})$ has very many local minima. Thus computations should be made for different initializations for \mathbf{x} to find a good local or the global minimum. For the case study

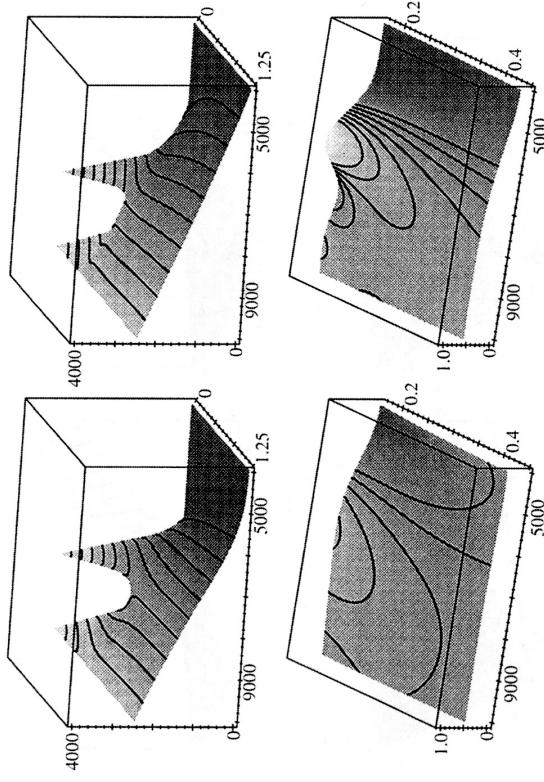


Fig. 4. Black-Scholes-value of an optimized European double-barrier DAX call ($\Delta_{aoEdbc} \approx 0.5$, strike 7000) dependent on the DAX in [4000, 10000] and the maturity period in [0, 1.25] years for an implied DAX volatility of 0.17 (upper left) and 0.45 (upper right) and Δ_{aoEdbc} of this call dependent on the DAX and the implied DAX volatility in [0.17, 0.45] for a maturity period of 1.25 years (lower left) and 0.417 years (lower right). Contour lines represent $\Delta_{aoEdbc} \equiv 0.35$, $\Delta_{aoEdbc} \equiv 0.40$, \dots , $\Delta_{aoEdbc} \equiv 0.65$.

and $\Delta_{opt} = 0.5$ the best vector found is

$$\mathbf{x}^* = (3000.00, 688.459, 865.610, 3000.00, 2433.73)^T \quad (11)$$

see Fig. 4 and compare Fig. 4 to Figs. 2 and 3. The insurance company can sell optimized DAX calls at $t = 0$ with $\Delta \in [0.38, 0.46]$ dependent on the actual implied DAX volatility $\sigma \in [0.17, 0.45]$ (covered call writing), see Fig. 4. The company accepts the obligation to pay the cash settlement (7), (8) or (9) at the option's expiration. In comparison to the plain vanilla DAX call the optimized DAX call has a widely almost constant $\Delta_{aoEdbc} \approx \Delta_{opt}$. Both buyer and seller of the optimized DAX call can benefit from this desirable behaviour. The actual Beta-version of the WARRANT-PRO-2 software handles an user definable number of parameters x_i to discretize the initial/boundary conditions (7) and (9). Computations with up to 25 parameters for (7) and up to 10 parameters for (9) have converged. However, computing times are very long and the large number of bad local minima makes it difficult to find good optimized European double-barrier DAX calls. So far it turned out that the optimal $C_{up}^*(t)$ tends to be almost constant and that the optimal

$C_T^*(p)$ tends to be more and more oscillating for an increasing number of parameters.

5 Outlook

An user friendly Windows/Linux version of WARRANT-PRO-2 with graphical user interface is under development in cooperation with the Dresdner Bank AG, Frankfurt. The software will provide a framework for professional option and warrant design. The gradient of $\varepsilon(\mathbf{x})$ needed for the minimization of $\varepsilon(\mathbf{x})$ currently is computed via numerical differentiation. The gradient's accuracy is only 2 – 3 correct decimal digits due to the low accuracy of $\varepsilon(\mathbf{x})$ (4 – 7 digits). The gradient computation will be implemented with the help of automatic/algebraic differentiation (accuracy 4 – 6 digits), see Griewank (2000). Additionally the improved, analytic gradient computation will cut down the computing times significantly. Moreover, ongoing research of the first author is dedicated to the replacement of the deterministic Black-Scholes-model by more accurate heuristic option pricing models, see Breitner (2000).

Acknowledgements

The authors gratefully appreciate support by Prof. Dr. P. E. Gill, University of California San Diego, providing his excellent SQP optimization method NPSOL, by the Market Maker Software AG, Kaiserslautern, and by the Dresdner Bank AG, Frankfurt.

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